CHAPTER 3

Seeing in a weird light: relativity

Key words

special relativity, general relativity, inertial frame of reference, invariant, principle of relativity, fictitious forces, Maxwell's equations, electromagnetic wave, mechanical medium, luminiferous aether, interferometry, beam splitter, aether drag, Michelson–Morley experiment, null result, postulate, simultaneity, Lorentz factor, time dilation, proper time, twin paradox, proper length, length contraction, rest mass, proper mass, relativistic mass, spacetime, mass-energy.

Just some minor problems

You may have heard it said that some physicists think that a 'theory of everything' is just around the corner. This attitude is not new. Many physicists thought this about what is now called 'classical Newtonian physics', towards the end of the late 19th century. There were just a few minor problems with understanding the way light travels through space that needed to be fixed, and then the job of physics would be finished.

Well, those 'minor problems with light' led to the twin pillars of modern physics: quantum mechanics and relativity. Einstein was a key player in both, especially relativity, which comes in two parts. **Special relativity** replaced Newton's mechanics and the later **general relativity** replaced Newton's universal gravitation. A century later, relativity still defies common sense, bending space, time and the mind, but it has not yet failed any experimental test. In this chapter we will only deal with the theory of special relativity.

'Common sense is nothing more than a deposit of prejudices laid down by the mind before you reach eighteen.'

A Einstein

Web Destinations

3.1 Frames of reference and classical relativity

Outline the nature of inertial frames of reference.

Discuss the principle of relativity.

Before we talk about relativistic weirdness, recall **inertial frames of reference** (see *in2 Physics @ Preliminary* section 3.3). Inertial frames of reference are required for observers using Newton's laws or the Galilean transformation formula for relative velocity (see section 1.1). An inertial frame is *any* non-accelerating (including non-rotating) reference frame. Inertial frames can move at a constant velocity relative to each other.

All velocities are relative. There are *no* absolute velocities and there is no special absolutely stationary inertial frame. The classical Newtonian laws of mechanics and gravity are unchanged (or **invariant**) when transforming from one inertial frame to another (even though the *values* of some measurements such as velocity might change). This is called the **principle of relativity**.

To transform a situation from one inertial frame to another, you simply apply the Galilean transformation formula to all the velocities. You've already seen an example of this in the gravity assist example (Figure 2.4.2).

Interactive Module

Because there is no special inertial frame, no experiment purely within your own frame can detect the velocity of your frame, so absolute velocity is meaningless. You can only compare your frame's velocity *relative* to others. An example of this is waiting to depart in a train, looking out the window (Figure 3.1.1) to see that a train next to you is moving slowly away, only to find a few seconds later that, in fact, relative to the station it is your train that is moving. Your acceleration (including vibrations) was negligible—you felt no effect of your uniform velocity.

However, you *can* feel the *acceleration* of a non-inertial reference frame, and measure it using an accelerometer. The simplest accelerometer is a pendulum. If a pendulum hangs vertically in a car, your horizontal acceleration is zero. If you are accelerating horizontally, the pendulum will hang obliquely (Figure 3.1.2).

If you are observing from within a *non*-inertial (accelerating) frame, Newton's laws appear to be violated. Objects can appear to change velocity without a true net external force; in other words, you experience **fictitious forces** or pseudo-forces (see *in2 Physics @ Preliminary* p 39). For example, in a car taking a corner, you experience the sensation of being 'thrown outwards' by a fictitious centrifugal force. If viewed from the inertial frame of the footpath, you evidently are pulled inwards by a true centripetal force. (We've cheated a bit. The Earth is turning, so the footpath is not strictly an inertial frame. However, Earth's radius is so large that in most human-scale situations, fictitious forces due to Earth's rotation are negligible.)

Another view of tests for non-inertial reference frames is that they involve detecting fictitious forces. It's a two-step process. First, analyse an object within that frame of reference and decide what true external Newtonian forces must act on the object. Then, look for apparently 'extra' or 'missing' forces—evidence of a non-inertial frame. For example, judged from the inertial frame of the ground, the downward weight $m\mathbf{g}$ and the upward normal force $\mathbf{N}$ of the seat are the only true forces on an astronaut during launch. Within the accelerating rocket, the sensation of enhanced weight (downwards) associated with g-force has the same magnitude as $\mathbf{N}$ but is apparently in the wrong direction and is therefore fictitious.

A pendulum accelerometer hangs obliquely within an accelerating car as though there is a fictitious horizontal component of weight. In free-fall (or orbit), the apparent *absence* of weight is also fictitious. Your frame accelerates downwards, so true weight becomes undetectable to you, as though your true downward weight is cancelled by a fictitious upward gravity. The effects of neither 'force' show up separately on an accelerometer.

Figure 3.1.1 Who is *really* moving?

PHYSICS FOR FUN

TRY THIS!

FICTITIOUS FUN

While sitting on a playground merry-go-round with a friend, try playing 'catch' with a slow moving tennis ball. The fictitious centrifugal and Coriolis forces will 'cause' the ball to appear to follow warped trajectories, making it difficult to catch.

Worked example

QUESTION

A Christmas decoration is hanging obliquely inside your car, 5° from vertical and pointing towards the car's left side. **Describe** quantitatively the car's motion (no skidding!).

SOLUTION

Only two true external forces act on the decoration: tension and weight (Figure 3.1.2). Because there is an angle between them, they aren't 'equal and opposite', so the decoration experiences a net real force and acceleration sideways (in this case centripetal). The net force and acceleration point towards the right side of the car, so the accelerometer (and the car) is steering towards the right.

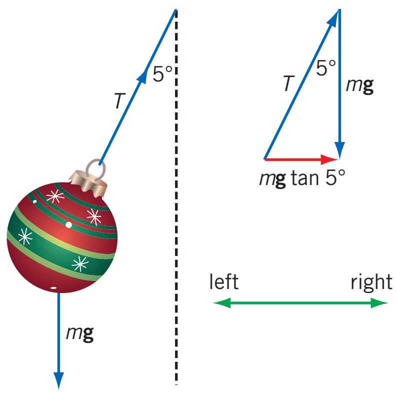

Figure 3.1.2 Festive season pendulum accelerometer

The 'centrifugal force' perceived by the occupants of the car to be pulling the decoration toward the left side of the car is fictitious.

From Figure 3.1.2, the magnitude of the centripetal acceleration is:

$$a_c = \frac{F_c}{m} = g\tan 5° = 9.80 \times 0.0875 = 0.86 \text{ m s}^{-2}$$

PRACTICAL EXPERIENCES
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Perform an investigation to help distinguish between non-inertial and inertial frames of reference.

Note: This test is *subjective*—it requires personal judgement (hence possible bias). No measurement alone can identify a force as fictitious. For example, no pure measurement can tell the difference between true weightlessness and the fictitious weightlessness of free-fall. You can only tell the difference by looking down and seeing the Earth below; judgement says there *should* be gravity, but you can't feel it. The inability of measurement alone to distinguish the effects of true gravity from the effects of g-force is what Einstein used as the starting point for his re-writing of the law of gravitation in his theory of general relativity, but you'll have to wait until university physics to learn about that!

This approach to distinguishing between inertial and non-inertial frames relies on a classical concept of force. In Einstein's relativity, the concept of force is more complicated and is used much less.

The term *fictitious force* doesn't mean the observed effects are imaginary, as the victims of a cyclone or astronauts who are subject to g-force can attest. It simply means that the apparent force doesn't fit Newton's definition of a true force. It is always possible to re-analyse fictitious forces using an inertial frame and to account for all observed effects using only true Newtonian forces.

PHYSICS PHILE

FICTITIOUS CYCLONE? YEAH RIGHT!

Effects associated with so-called fictitious forces of Earth's rotation are not always negligible. The Coriolis force is a fictitious tangential force appearing in rotating frames of reference and is associated with the formation of cyclones.

Figure 3.1.3 Satellite photo of a cyclone

CHECKPOINT 3.1

1. **Define** an inertial reference frame.
2. **Recall** the Galilean transformation formula for relative velocities.
3. **Outline** why we usually treat the Earth as an inertial frame, given that it is rotating.
4. **Discuss** whether or not centripetal force is fictitious.
5. In free-fall, you don't experience any extra apparent forces. Are you in an inertial frame? **Explain**.
6. What apparatus would distinguish true weight from apparent weight due to g-force ?
7. The values of some measurements such as velocity might change, but the laws of mechanics are the same in all frames of reference. True or False? **Explain**.

3.2 Light in the Victorian era

The 19th century was a period of enormous advance in the study of electricity and magnetism. Faraday, Ampere, Oersted, Ohm and others, through theory and experiment, produced a large collection of equations and phenomena. There were hints of connections between electricity and magnetism—an electrical current can produce a magnetic field (see *in2 Physics @ Preliminary* section 12.3) and a changing magnetic field can induce a changing electric field or current (section 4.1).

The Scottish theoretical physicist, James Clerk Maxwell (1831–1879) collected the existing equations to reduce them down to the minimum number. He reduced them down to eight equations (which expanded to 20 when he included all the *x*-, *y*- and *z*-components). A self-taught electrical engineer called Oliver Heaviside (1850–1925), using the newly developed mathematics of vectors, reduced Maxwell's equations to four. We now call those four equations **Maxwell's equations**.

It puzzled Maxwell that his equations were almost symmetrical in their treatment of electrical and magnetic fields—almost but not quite. So he added a term to his equations, assuming that a changing electrical field can induce a magnetic field (not previously observed). When he did this, he showed that an oscillating magnetic field would induce an oscillating electric field and vice versa, resulting in a self-sustaining **electromagnetic wave**. From his equations he calculated the speed of that wave to be equal to the speed of light in a vacuum (which is now called c and equals $2.998\,792\,458 \times 10^8$ m s^{-1}).

It was either an astonishing coincidence or strong circumstantial evidence that light is an electromagnetic wave (see *in2 Physics @ Preliminary* p 84). Heinrich Hertz (1857–1894) experimentally confirmed the predicted speed and properties of these electromagnetic waves.

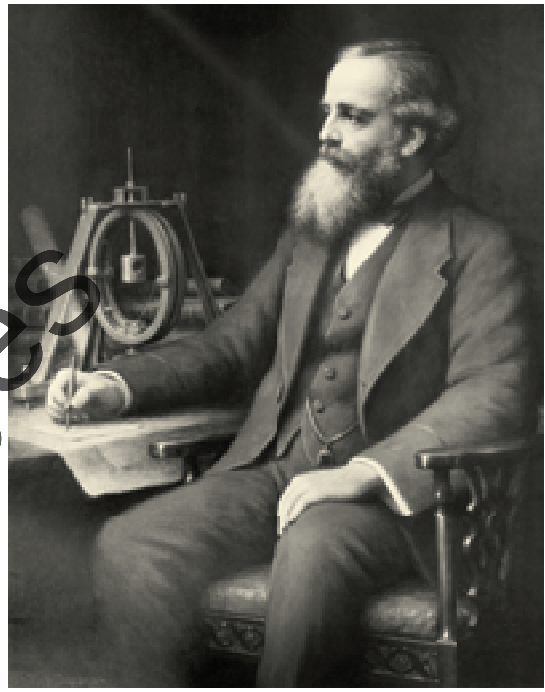
Figure 3.2.1 James Clerk Maxwell

What is light's medium?

Outline the features of the aether model for the transmission of light.

Until then, every existing kind of wave needed a **mechanical medium**; for example, sound propagates through air, earthquakes through rock, musical vibrations along a violin string, ripples along water and so on (see *in2 Physics @ Preliminary* section 5.3). To sustain a wave, a medium needs two properties: resilience (or stiffness) and inertia (any density- or mass-related property). The higher the stiffness and the lower the inertia, the higher the wave speed.

It was assumed that light also needs a medium, which was called **luminiferous aether** or just aether (US spelling: ether). Luminiferous means 'light-bearing', and 'aether' was the air breathed by the gods of Greek mythology.

So Maxwell developed a model for aether, assigning it bizarre mechanical properties consistent with the behaviour and enormous speed of light. It needed to be far less dense than air but much stiffer than any known material. Despite its stiffness, aether was assumed to penetrate all materials effortlessly. Conversely, it needed to be able to be penetrated without resistance by all objects that move freely through space, including Earth hurtling around the Sun.

If you shout with the wind blowing behind you, then, relative to you, the velocity of sound would be higher than if the air were still. This is because the velocity of sound (and other mechanical waves) is the sum of its velocity relative to the medium and the velocity of the medium itself. In other words, mechanical waves seem to obey the Galilean transformation. It was assumed that light should also obey it, so the speed of light should be affected by the motion of the aether.

However, Maxwell's equations appeared to allow only one particular value for the speed of light in a vacuum. The Galilean transformation and Newton's laws imply it is *impossible* for the speed of light to appear to be the same to all observers with different relative speeds. Perhaps the speed specified by Maxwell's equations is the speed relative to the aether only. However, this meant that the aether represented a preferred reference frame for Maxwell's equations, which was inconsistent with the classical principle of relativity.

M and M

Describe and evaluate the Michelson–Morley attempt to measure the relative velocity of the Earth through the aether.

Discuss the role of the Michelson–Morley experiments in making determinations about competing theories.

Given that the Earth was supposed to be hurtling around the Sun, through the aether at 3×10^4 m s^{-1}, the resulting 'aether wind' (or aether drift) relative to Earth should affect measurements of light speed differently according to the time of day and time of year as the Earth rotated and orbited the Sun, changing its orientation relative to the aether.

So in the 1880s, the experimentalist Albert Michelson (1852–1931), joined later by Edward Morley (1838–1923), attempted to measure changes in the speed of light throughout the day due to this shifting aether wind. They used a very sensitive method called **interferometry** (see section 21.5), which Michelson had used some years earlier to accurately measure the speed of light.

Recall constructive and destructive interference (see *in2 Physics @ Preliminary* p 102 and p 126). If two light beams are projected onto a screen, then a bright 'fringe' occurs at places where the two beams are in phase (constructive interference). Where they are out of phase, destructive interference results in a dark fringe. The pattern of bright and dark fringes is called an interference pattern (Figure 3.2.2).

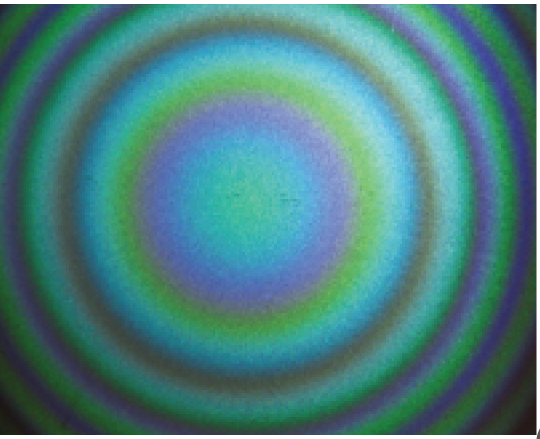

Figure 3.2.2 Interference pattern in a Michelson interferometer illuminated by a mercury vapour lamp. Patterns of different shapes (such as vertical bands) are possible and depend on exactly how the interferometer is aligned.

Interference turns a pair of monochromatic (single wavelength) light beams into an extremely sensitive ruler for which the interference fringes are like magnified ruler markings one light wavelength apart. For visible light, this spacing is less than 8×10^{-7} m and corresponds to time intervals of less than 3×10^{-15} s. If the two light beams travel via different paths, then a very small change in the length of one path will change the relative phase, resulting in a detectable change in the position of fringes in the interference pattern. A change in wave speed along one of those paths should have a similar effect on phase.

PRACTICAL EXPERIENCES
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Michelson and Morley set up an interferometer in which the light was divided into two perpendicular beams or 'arms' by passing it through a half-silvered mirror or **beam splitter** (Figure 3.2.3). The apparatus was built on a heavy stone optical bench floating in mercury, to allow rotation and damp out vibrations. They assumed that if one interferometer arm was pointing parallel to the aether wind, the speed of light should be slightly different in the two arms. The time of flight of the light in the arm parallel to the aether wind should be slightly longer than that of light along the perpendicular arm. As the Earth (or the apparatus) rotates, this speed difference, as measured by the positions of the interference fringes (Figure 3.2.2), should change with the angle.

Gather and process information to interpret the results of the Michelson–Morley experiment.

Figure 3.2.4 summarises the classically predicted effect of aether wind on the resultant light speed in the two arms of the interferometer. Let's calculate the expected time difference. Suppose the total distance from beam splitter M_S to M_1 (or M_2) is L, then the round-trip for each arm is $2L$.

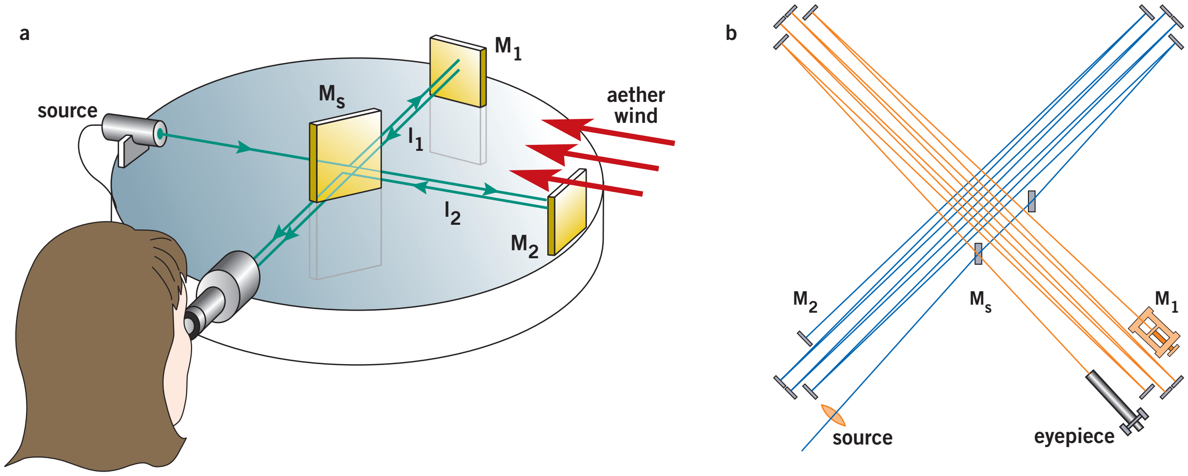

Figure 3.2.3 The Michelson–Morley interferometer drawn as (a) a simplified schematic and (b) an actual ray diagram. Multiple reflections were used to make the effective length of the arms very long hence more sensitive to changes in light speed. M_S is the half-silvered beam splitter mirror.

In the arm perpendicular to the aether wind (speed v), if c is light speed relative to the aether, then the resultant light speed is $\sqrt{c^2 - v^2}$ (Figure 3.2.4a) and the time taken for light to do a round-trip is:

$$t_1 = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c} \times \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In the arm parallel to the aether wind (speed v), for half the trip against the wind, the speed of light would be $c - v$, and for the other half with the wind it would be $c + v$, so the time taken would be longer:

$$t_2 = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2L}{c} \times \frac{1}{1 - \frac{v^2}{c^2}}$$

(Check that you agree that since v is smaller than c, time in the parallel arm t_2 is longer than time in perpendicular arm t_1.)

Other factors such as thermal expansion or contraction of the apparatus could cause apparent drift in the interference pattern, but the shift due to rotation of the apparatus (or the Earth beneath it) would be a sine wave with a period equal to the rotation period of the apparatus, so any drift not due to rotation could be detected and subtracted. Michelson and Morley graphed the position of interference fringes versus rotation angle at different times of the day, but concluded that the small observed shifts could be explained as drift in the experimental apparatus. Over several years, scientists repeated the measurements, with some reports of possible changes in interference over the day; but eventually the consensus was that any observed effect was well below what was expected by the aether theory and could be explained by drift in the apparatus.

George Fitzgerald and Hendrik Lorentz attempted to squeeze the Galilean transformation into Maxwell's equations, concluding that charged particles (such as charges in atoms) moving through the aether with speed of v must shrink in the direction of motion by a factor of $\sqrt{1 - v^2/c^2}$. The interferometer arm parallel to the aether wind would shrink just enough to compensate for the change in light speed and hence cancel the expected change in the interference pattern.

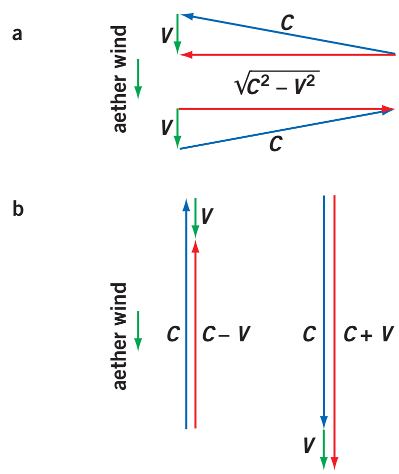

Figure 3.2.4 Classical effect of aether wind on light velocity. (a) The resultant velocity perpendicular to the wind and (b) resultant velocity parallel to the wind. Blue = light velocity relative to aether, green = aether velocity and red = resultant light velocity

Another reason suggested for failure to see the shift was that perhaps aether only penetrates transparent objects, so aether was trapped by large opaque mountains and valleys or buildings and dragged along by the moving Earth, similar to the way in which air is trapped in the fur of a running dog. A flea conducting scientific experiments on the dog's skin would be unaware that outside the fur, air is whooshing backwards relative to the dog. This idea was called **aether drag**. If this were true, then at the tops of mountains, closer to outer space, the aether wind might be detectable. Some experimenters repeated the experiment on mountains without success (apart from a controversial partial result).

Failure to detect undeniable effects of aether wind caused some physicists to question if it even existed. Maxwell's equations only mention electric and magnetic fields. The aether is not required by the equations. Einstein assumed it didn't exist, but said that relativity was not an attempt to explain Michelson and Morley's negative result, but rather, he was motivated by the properties of Maxwell's equations. However, in physics, when experiment and theory say the same thing, you're probably on the right track. Today almost all physicists agree that there is no aether.

The **Michelson–Morley experiment** is often called 'the most famous failed experiment'. It was not exactly a 'failure'. In 1907, Michelson was awarded the Nobel Prize for Physics for his work. The result of an experiment that fails to find evidence of an expected effect despite careful design and execution is more correctly called a **null result**. This null result was one of the most important in the history of physics, because it helped bring about a whole new way of seeing the universe.

CHECKPOINT 3.2

1. **Describe** Maxwell's circumstantial evidence that light is an electromagnetic wave.
2. **Discuss** why it was assumed that light required a medium or 'aether' to propagate in.
3. Maxwell's equations predicted that the speed of light should depend on the speed of the medium, but this was contradicted by the Michelson–Morley experiment. True or false? **Explain**.
4. In the classical analysis of the Michelson–Morley interferometer, which arm required the longer time of flight?
5. Is it correct to say that the Michelson–Morley experiment didn't show any change in the interference pattern? **Explain**.
6. **Outline** how Fitzgerald and Lorentz explained the apparent absence of evidence for aether wind.
7. **Describe** aether drag.
8. **Discuss** which played a greater role in motivating Einstein's work, the work of Michelson and Morley or that of Maxwell.

Explain qualitatively and quantitatively the consequence of special relativity in relation to:

- the relativity of simultaneity
- the equivalence between mass and energy
- length contraction
- time dilation
- mass dilation.

3.3 Special relativity, light and time

Although relativity is Einstein's theory, many of the underlying ideas or mathematical formulae were inspired or anticipated by others including Poincaré, Lorentz and Minkowski. Einstein, being a theoretician, did not conduct laboratory experiments. However, he is famous for making good use of the 'Gedankenexperiment' or 'thought experiment' to boil abstract ideas down into simple concrete ones. Theory can sometimes be derived by imagining an experiment being done, even if it is impractical. We will mention some of his thought experiments in this section.

Speed of light

Newton regarded space and time as absolute. In practical terms, this means that the length of 1 metre, the duration of 1 second and the geometric properties of shapes would be the same to all observers everywhere. Not all physicists agreed, but the success of Newton's laws silenced any philosophical discussion.

However, Maxwell's theory (and the Michelson–Morley experiment) pointed to the speed of light in a vacuum being constant to all observers. So Einstein said one of three things must be wrong: the principle of relativity (the invariance of laws of mechanics in all inertial reference frames), Maxwell's equations or the Galilean transformation (the basis of all of classical mechanics).

The principle of relativity seemed very fundamental to Einstein, so he didn't reject that. In fact, he extended Galileo and Newton's principle of relativity to include *all* laws of physics, not just mechanics. He called it his first **postulate**.

Following a suggestion by Jules Henri Poincaré (1854–1912), Einstein decided that as the speed of light in a vacuum was invariant in all inertial frames, then that must also be a law of nature, which he called the *postulate of the constancy of the speed of light*.

Maxwell's equations accurately described electromagnetic phenomena, so Einstein didn't want to reject them. So it must be the Galilean transformation (and hence all of classical mechanics) that was wrong. But it is difficult to see how something so simple could possibly be wrong.

Suppose you are on a moving train, shining a torch towards the front of the carriage. To your eyes, the light travels the length of the carriage L. To you, its speed is the length of the carriage divided by the time t it took to get there $c = L/t$. To an observer at the train station, the light travelled the length of the carriage plus the distance D the carriage travelled in that time: $c = (L + D)/t$. The arithmetic is so laughably simple. How could both observers *possibly* get the same value for c? It could only be possible if you and the observer at the train station disagree on the lengths L or D or the time period t. In other words, if the speed of light is constant then length (space) and/or time are not absolute—they *must* depend on the state of motion of the observer.

So why had no-one noticed until 100 years ago? Classical mechanics had successfully described phenomena for three centuries, but it had never been tested for things moving at close to the speed of light. Classical mechanics and the Galilean transformation are accurate approximations at speeds well below the speed of light. Only when the properties of light itself were examined, did the problems become obvious.

Einstein showed (in several ways), that the speed of light is the ultimate speed limit—no observer can reach the speed of light. As a teenager, he asked 'What would the world look like if I rode on a light beam?' He answered as an adult with a thought experiment. A light beam is a wave of oscillating electric and magnetic fields moving at the speed c. If you were in the same reference frame as the light beam, you would observe stationary electric and magnetic fields that vary as sine waves in space, but are constant in time. This is not an allowable solution to Maxwell's equations, so it is not possible for an observer to travel at the speed of light—it is the ultimate speed limit.

Simultaneity

Einstein demonstrated that **simultaneity** is relative. Events apparently simultaneous to one observer are not necessarily so to all observers. Let's use Einstein's own

Describe the significance of Einstein's assumption of the constancy of the speed of light.

Identify that if c is constant then space and time become relative.

PHYSICS PHILE

WHAT'S SO SPECIAL ABOUT RELATIVITY?

Einstein called his 1905 replacement theory for Newton's mechanics *special relativity*. It is a 'special case' in the mathematical sense of being restricted to particular conditions—to inertial reference frames. Einstein's *general relativity* came 11 years later and was generalised to include non-inertial reference frames. It replaced Newton's gravity.

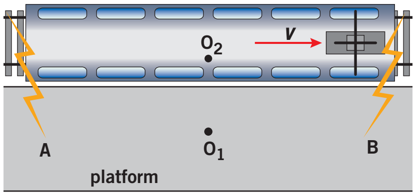

Figure 3.3.1 Lightning strikes at A and B appear simultaneously to observer O_1 but not to O_2.

thought experiment. Observer O_1 is standing on a train station platform equidistant from points A and B, which appear to O_1 to be struck simultaneously by two bolts of lightning (Figure 3.3.1). Because the light travelled the same distance from both points and the light reached O_1 at the same time, O_1 judges that the lightning bolts struck simultaneously.

Suppose observer O_2 is sitting in a high speed train passing the platform without stopping. O_1 calculates that at the moment the lightning struck, O_2 on the train was also equidistant from A and B and so naively assumes that O_2 would also see the events as simultaneous. However, by the time the light reached O_1's eyes, O_2 was closer to B and had already seen the light from B but not A, and concluded B was struck first. Only if two events occur simultaneously at the same *place*, will *all* observers agree that they were simultaneous.

Similar arguments can show that the *order* of events is relative. Since an effect must come after its cause, relativity places restrictions on possible chains of cause and effect. Because the speed of light is the universal speed limit, two events separated by distance cannot have *any* influence over each other over a time scale shorter than the time required for light to travel between them. Because *no signal or influence* can travel between a cause and its effect faster than light, apparent changes in the simultaneity or order of events based on the passage of light is more than just an optical illusion—it represents a fundamental limitation of reality.

Solve problems and analyse information using: $E = mc^2$

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Analyse and interpret some of Einstein's thought experiments involving mirrors and trains and discuss the relationship between thought and reality.

Time dilation

The relativity of simultaneity suggests that time itself is a bit rubbery. Relativity predicts that the rate of passage of time differs, depending on the velocity of the observer.

Consider the following thought experiment (Figure 3.3.2). Suppose you (observer 0) are on a train, moving with speed v relative to the ground, while measuring the speed of light by shining a light pulse vertically towards a mirror on the train ceiling, a distance D from the light source. Outside on the ground is observer v who appears to you to be rushing past with horizontal speed v and watching your experiment. Both you and observer v regard your own reference frame as stationary. However, you both agree on three things: (1) the speed of light, (2) your relative horizontal speed v and (3) the height of the mirror D. You both agree on the height of the mirror because you are both in the same reference frame with respect to vertical components.

Your light source also contains a detector capable of timing the interval t_0 between the emission and detection of the light pulse. You do the experiment and use the speed formula $c = 2D/t_0$ to obtain the correct speed of light.

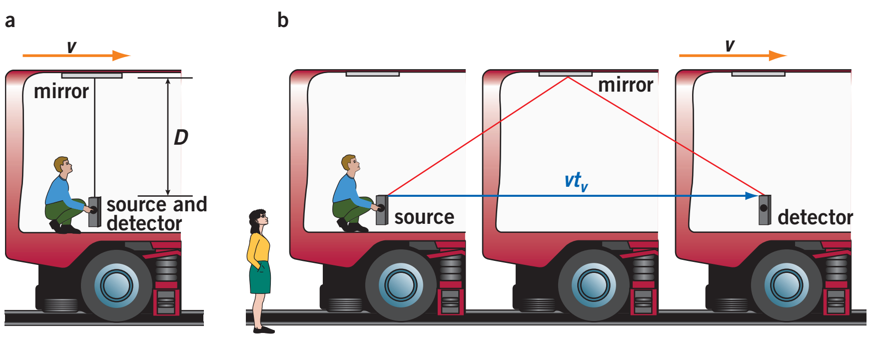

Figure 3.3.2 Measuring the speed of light on a train as seen by (a) observer 0 within the train and (b) observer v from the reference frame on the ground

Observer v disagrees about what happened. She was carrying an accurate stopwatch and timed the event independently, getting a longer time interval of t_v. Using Pythagoras' theorem, she calculates that the path length of the light (Figure 3.3.2b) was not $2D$, but rather:

$$2\sqrt{D^2 + (\tfrac{1}{2}vt_v)^2} = \sqrt{4D^2 + (vt_v)^2}$$

She agrees on the speed of light c so:

$$c = \frac{\sqrt{4D^2 + (vt_v)^2}}{t_v}$$

Square and rearrange: $c^2t_v^2 = 4D^2 + (vt_v)^2$ but observer 0 says $c = 2D/t_0$

Eliminate D: $c^2t_v^2 = c^2t_0^2 + (vt_v)^2$

Rearrange: $t_v^2(c^2 - v^2) = c^2t_0^2$

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The factor $1/\sqrt{1 - v^2/c^2}$ is called the **Lorentz factor** (abbreviated as γ). It is always larger than 1, which means that if t_0 is the time between ticks on observer 0's clock then observer v will see observer 0's clock whiz by at speed v, ticking more slowly with a time t_v between ticks. This is **time dilation**. The word *dilation* means 'spreading out', just like the time between clock ticks.

An observer moving relative to you and observing a clock ticking (or any series of events) stationary in your frame, will judge events to happen more slowly than you observe them. Note that the dilation is only observed in clocks in *other* frames of reference. You can't observe time dilation in clocks in your own frame, no matter how fast *others* think you're moving.

As there is no preferred inertial frame, the effect is symmetrical; that is, observers can be swapped. You both agree on your relative speed v but both insist the *other* observer is moving and their clock is running slow. You are both right because time is relative.

A time interval observed on a clock that is stationary relative to the observer is called the **proper time** for that reference frame. To generalise this idea, t_0 can represent the time between any two events (such as the ticks of a clock) that occur in the *same place* in the frame of the observer. If the events are separated in space, then extra time is required to allow for light to travel between the positions of the two events.

Global positioning system (GPS) receivers estimate your position by measuring how long it takes the GPS signal to travel from the satellites to your receiver. The orbital speed of the satellites is large enough that the calculation needs to take time dilation into account. (It also takes into account a larger effect from general relativity: time runs more slowly in a stronger gravitational field.)

The Lorentz factor γ approaches infinity as speed approaches the speed of light (Figure 3.3.3). This means that time in a frame of reference approaching the speed of light (relative to the observer) will come to a complete stop. In other words, nothing can be seen to happen in such a frame, which is another reason why the speed of light is the ultimate speed limit.

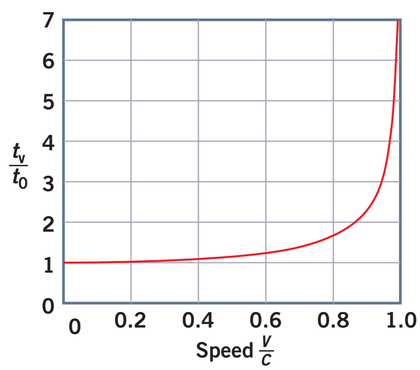

Figure 3.3.3 Plot of the ratio of a time interval in a moving reference frame to proper time (t_v/t_0) versus speed in units of c. Note that at speeds below ~0.1c, the ratio ≈ 1, so time behaves nearly classically.

PHYSICS PHILE

MUON AND ON AND ON

Fast-moving muons are produced in the upper atmosphere by cosmic ray bombardment. Time dilation extends their normally short lifetimes long enough to allow many of them to make it to Earth, where they are a significant component of Earth's background radiation.)

Figure 3.3.4 Both twins think the other's clock is moving slower, but who is older at the end of the journey?

Worked example

QUESTION

A muon is like a heavy electron, and at low speed it decays with a mean lifetime of 2.2×10^{-6} s. Suppose a beam of muons is accelerated to 80% of the speed of light. What would their mean lifetime be in the laboratory reference frame?

SOLUTION

Lifetime in the muon's frame: $t_0 = 2.2 \times 10^{-6}$ s

Speed of muon's frame: $v = 0.80c$

Lifetime in the laboratory frame is t_v: $$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.2\times10^{-6}}{\sqrt{1-0.80^2}} = 3.7\times10^{-6} \text{ s}$$

The twin paradox

When two people pass by quickly, observing each other, they both think the other's clock is running slower. The principle of relativity says you are both right. The **twin paradox** is a thought experiment in special relativity. Bill goes for an intergalactic cruise travelling at close to the speed of light (in Earth's frame), while Phil stays on Earth (Figure 3.3.4). During the flight, they both *correctly* conclude that the *other* twin's frame is moving, and so he is ageing more slowly.

But what happens when Bill comes back home? Observations in the same frame should agree. It turns out that Bill is younger than Phil. Does this violate the principle that all inertial frames of reference are equivalent? No. Bill turned around (accelerated) to come home. The situation is no longer symmetrical. Special relativity isn't enough to explain what Bill saw from his accelerating frame (he needs general relativity). However we have no difficulty talking about what (non-accelerating) Phil saw.

By turning around and coming back, Bill left his original inertial frame and re-entered Phil's frame, so he should agree with Phil. Phil remained in his inertial frame all along, so his conclusions (that Bill was moving and so is younger) have been consistent with special relativity throughout and, in his frame, correct. If instead Phil had hopped into another craft and caught up with Bill's inertial frame, then Bill's original conclusion would have been correct and Phil would have been younger.

This prediction has been confirmed using highly precise, twin atomic clocks and an aeroplane.

CHECKPOINT 3.3

1. State Einstein's first postulate and its alternative name.
2. State Einstein's second postulate and its alternative name.
3. **Outline** why Newton's classical mechanics is so successful despite a fundamental error (the Galilean transformation).
4. **Explain** why the speed of light places restrictions on possible chains of cause and effect.
5. Write the formula for time dilation.
6. A clock moving towards you appears to slow down. If the clock were moving in the opposite direction, would it speed up?
7. What is the name given to a time interval measured on a clock that is stationary in your frame of reference?
8. In the twin paradox, during a period of constant relative motion, both Bill (astronaut) and Phil (earthling) observe the other twin's watch ticking more slowly. Who's observation is *actually* correct?

3.4 Length, mass and energy

The formula for time dilation has already upset our common sense. However, once the clocks start talking to the rulers and the masses, things can only get more bizarre.

Solve problems and analyse information using: $E = mc^2$

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Explain qualitatively and quantitatively the consequence of special relativity in relation to:
- the relativity of simultaneity
- the equivalence between mass and energy
- length contraction
- time dilation
- mass dilation.

Length contraction

There is a grain of truth in Lorentz and Fitzgerald's suggestion (section 3.2) that the arm of a Michelson interferometer contracts by a factor of $\sqrt{1-v^2/c^2}$ in the direction of motion. Their formula was correct, but their interpretation that it resulted from motion through the (non-existent) aether was wrong. Also, the contraction doesn't happen in the frame of reference of the experimenter. Moreover, their hypothesis was 'ad hoc'; it was designed only to patch a hole in the old theory without resulting in any additional testable predictions. So Einstein re-interpreted their mathematics in light of his theory of relativity.

If an object is moving with speed v relative to the observer, the length of the object in the direction of that motion will be observed to be contracted according to the formula:

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

where l_0 is the length judged by an observer who is stationary relative to the object (**proper length**) and l_v is the length judged by an observer in a frame moving with speed v relative to the object. The **length contraction** only takes place in the dimension parallel to the motion. Just like time dilation:

1 the effect is symmetrical, which means the observers can be swapped—both insist it is the other person's ruler that is too short

2 you cannot observe a Lorentz contraction within your own frame.

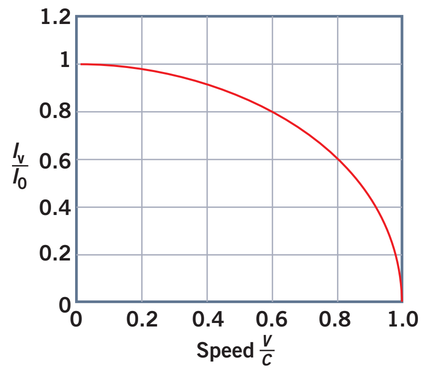

Figure 3.4.1 Plot of the ratio of length in a moving reference frame to proper length (l_v/l_0) versus speed in units of c. Note that as speed approaches c, l_v shrinks to zero—another reason why the speed of light is unattainable.

Imagine that observer 1 and observer 2 are trying to measure the length of a rod, but all they have is a stopwatch. They already know accurately (and agree on) their relative speed v. Observer 1 is holding the rod and observer 2 is holding the stopwatch. They whoosh past each other almost touching, both looking at the watch.

Observer 2 is stationary relative to the watch (Figure 3.4.2a), so he knows the reading on his watch is his proper time. As the rod passes by, the watch reads zero at the start of the rod and t_2 at the end, so the rod took a time t_2 to pass by. Therefore he calculates that the length of the rod in his frame is $l_v = vt_2$.

Observer 1 is stationary relative to the rod (Figure 3.4.2b), so she knows that its length for her is the proper length l_0. She agrees that the watch says t_2, but the moving watch seemed to be ticking too slowly, so the number on the watch *must* be too small. Using the time dilation formula, she calculates that the time t_1 in her frame was longer:

$$t_1 = \frac{t_2}{\sqrt{1-\frac{v^2}{c^2}}}$$

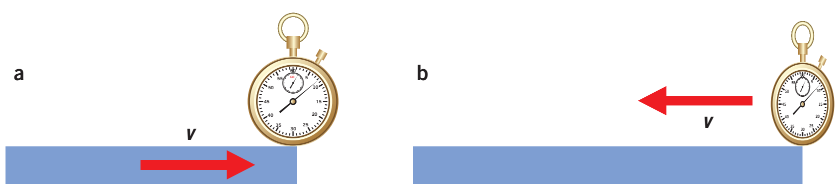

Figure 3.4.2 Measuring the length of a rod using a stopwatch as seen by (a) observer 2, holding the watch, and (b) observer 1, holding the rod

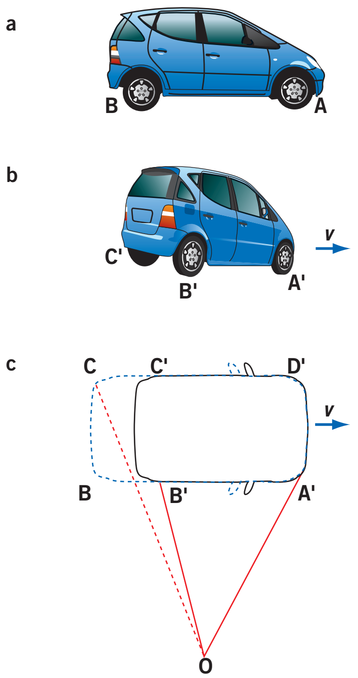

Figure 3.4.3 A fast-moving vehicle appears contracted horizontally, but also rotated away from the observer. The car is depicted when (a) stationary, (b) moving at high speed and (c) viewed from above. Corner C is normally out of sight, but at high speed, the vehicle moves out of the way fast enough to allow light reflected from C to reach your eyes at O, allowing you to see the car's back and side at the same time. This is called Terrell–Penrose rotation.

Observer 1 then calculates that the length of the rod is $l_0 = vt_1$ or

$$l_0 = \frac{vt_2}{\sqrt{1-\frac{v^2}{c^2}}}.$$

But observer 2 says that $l_v = vt_2$, so by substitution and rearrangement

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}.$$

Worked example

QUESTION

The distance travelled by light in one year, 9.46×10^{15} m, is called a light-year (ly). The nearest star to our Sun is Proxima Centauri, 4.2 light-years away.

Suppose you are travelling to Proxima Centauri at three-quarters of the speed of light.

a **Calculate** how long it takes to get there from Earth (measured using your on-board clock).

b **Discuss** whether this answer is a contradiction.

SOLUTION

a Both you and Earth-bound observers agree on your relative speed $0.75c$. In the spaceship's frame, the distance to Proxima Centauri is contracted:

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}} = 4.2\sqrt{1-0.75^2} = 2.78 \text{ ly}$$

$$t = \frac{l_v}{c} = \frac{2.78 \text{ ly}}{0.75c} = 3.7 \text{ years}$$

b 3.7 years is less than the 4.2 years that light takes to get there in Earth's frame. This is not a contradiction because in the spaceship's frame, light would only take 2.78 years because $l_v = 2.78$ ly.

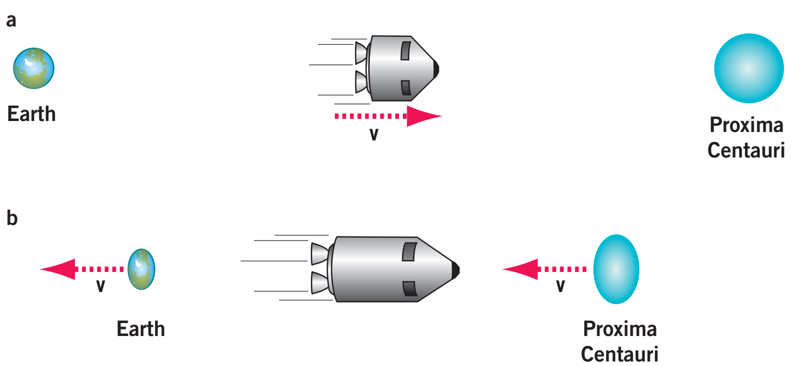

Figure 3.4.4 Trip to Proxima Centauri as seen by (a) earthlings and (b) the astronauts

Discuss the implications of mass increase, time dilation and length contraction for space travel.

Note that in the last example, the astronauts thought they experienced a short trip because the distance travelled was contracted, whereas the earthlings thought the astronauts felt their trip was short because their time had slowed.

Relativistic mass

If you measure the mass m_0 of an object at rest in your frame (**rest mass** or **proper mass**) and use the classical definition of momentum $\mathbf{p} = m_0\mathbf{v}$, then in collisions, momentum is not necessarily conserved for all reference frames. However, momentum *is* conserved if one instead uses $\mathbf{p} = m_v\mathbf{v}$ where m_v is the **relativistic mass**:

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

The relativistic mass of an object increases as its speed relative to the observer increases. As speed approaches c, the mass approaches infinity, so the force required to accelerate an object to the speed of light becomes infinite. This is yet another reason why the speed of light cannot be reached.

When accelerating particles in accelerators, this increase in mass needs to be taken into account, otherwise the machines won't work.

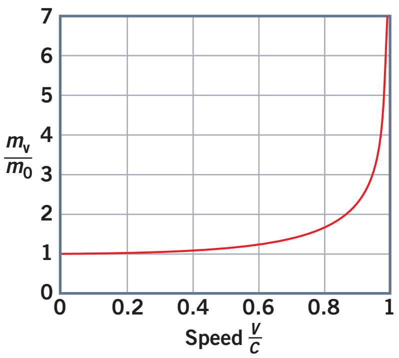

Figure 3.4.5 Plot of the ratio of relativistic mass m_v in a moving reference frame to rest mass m_0 versus speed in units of c. As speed approaches c, the relativistic mass approaches infinity.

Worked example

QUESTION

A medical linear accelerator (linac) accelerates a beam of electrons to high kinetic energies. These electrons then bombard a tungsten target, producing an intense X-ray beam that can be used to irradiate cancerous tumours. A typical speed for electrons in the beam is 0.997252 times the speed of light.

Calculate the Lorentz factor and hence the relativistic mass of these electrons, given the rest mass is 9.11×10^{-31} kg.

SOLUTION

$$\text{Lorentz factor } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.997252}} = 13.5$$

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = 9.11 \times 10^{-31} \times 13.5 = 1.23 \times 10^{-29} \text{ kg}$$

Note: When calculating Lorentz factors close to the speed of light, use a greater number of significant figures than usual, because you are subtracting two numbers of very similar size.

PHYSICS PHILE

RELATIVISTIC TRAIN CRASH

Trains A and B are about to collide head-on, each with a speed $0.5c$ relative to the station. So, relative to train B, train A is moving at the speed of light, right? Wrong! The replacement for Galileo's relative velocity rule in 1-dimension is:

$$\mathbf{v}_{(\text{A rel. to B})} = \frac{\mathbf{v}_A - \mathbf{v}_B}{1-\frac{\mathbf{v}_A\ \mathbf{v}_B}{c^2}}$$

The speed of train A relative to train B is:

$$\frac{0.5c-(-0.5c)}{1-\frac{0.5c\times(-0.5c)}{c^2}} = 0.8c$$

Solve problems and analyse information using:

$E = mc^2$

$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$

$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

Mass, energy and the world's most famous equation

The kinetic energy formula $K = \frac{1}{2}mv^2$ doesn't apply at relativistic speeds, even if you substitute relativistic mass m_v into the formula. Classically, if you apply a net force to accelerate an object, the work done equals the increase in kinetic energy. An increase in speed means an increase in kinetic energy. But in relativity it also means an increase in relativistic mass, so relativistic mass and energy seem to be associated. Superficially, if you multiply relativistic mass by c^2 you get $m_v c^2$, which has the same dimensions and units as energy. But let's look more closely at it.

PHYSICS FEATURE

TWISTING SPACETIME ... AND YOUR MIND

PFA 1. The history of physics

There are two more invariants in special relativity. Maxwell's equations (and hence relativity) requires that electrical charge is invariant in all frames. Another quantity invariant in all inertial frames is called the **spacetime** interval.

You may have heard of spacetime but not know what it is. One of Einstein's mathematics lecturers Hermann Minkowski (1864–1909) showed that the equations of relativity and Maxwell's equations become simplified if you assume that the three dimensions of space (x, y, z) and time t taken together form a four-dimensional coordinate system called *spacetime*. Each location in spacetime is not a position, but rather an *event*—a position and a time.

Using a 4D version of Pythagoras' theorem, Minkowski then defined a kind of 4D 'distance' between events called the spacetime interval s given by:

$$s^2 = (c \times \text{time period})^2 - \text{path length}^2$$
$$= c^2t^2 - ((\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2)$$

Observers in different frames don't agree on the 3D path length between events, or the time period between events, but *all* observers in inertial frames agree on the spacetime interval s between events.

Figure 3.4.6 One of the four ultra-precise superconducting spherical gyroscopes on NASA's Gravity Probe B, which orbited Earth in 2004/05 to measure two predictions of general relativity: the bending of spacetime by the Earth's mass and the slight twisting of spacetime by the Earth's rotation (frame-dragging)

In general relativity, Einstein showed that gravity occurs because objects with mass or energy cause this 4D spacetime to become distorted. The paths of objects through this distorted 4D spacetime appear to our 3D eyes to follow the sort of astronomical trajectories you learned about in Chapter 2 'Explaining and exploring the solar system'. However, unlike Newton's gravitation, general relativity is able to handle situations of high gravitational fields, such as Mercury's precessing orbit around the Sun and black holes. General relativity also predicts *another* wave that doesn't require a medium: the ripples in spacetime called 'gravity waves'.

How does this formula behave at low speeds (when v^2/c^2 is small)?

$$m_v c^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = m_0 c^2\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Using a well-known approximation formula that you might learn at university, $(1-x)^n \approx 1 - nx$ for small x:

$$m_0 c^2\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx m_0 c^2\left(1+\frac{1}{2}\times\frac{v^2}{c^2}\right) = m_0 c^2 + \frac{1}{2}m_0 v^2$$

Rearrange: $$m_v c^2 - m_0 c^2 = (m_v - m_0)c^2 \approx \frac{1}{2}m_0 v^2$$

In other words, at low speeds, the gain in relativistic mass $(m_v - m_0)$ multiplied by c^2 equals the kinetic energy—a tantalising hint that at low speed mass and energy are equivalent. It can also be shown to be true at all speeds, using more sophisticated mathematics. In general, mass and energy are equivalent in relativity and c^2 is the conversion factor between the energy unit (joules) and the mass unit (kg). In other words:

$$E = mc^2$$

where m is *any* kind of mass. In relativity, mass and energy are regarded as the same thing, apart from the change of units. Sometimes the term **mass-energy** is used for both. m_0c^2 is called the rest energy, so even a stationary object contains energy due to its rest mass. Relativistic kinetic energy therefore:

$$m_v c^2 - m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2$$

Whenever energy increases, so does mass. Any release of energy is accompanied by a decrease in mass. A book sitting on the top shelf has a slightly higher mass than one on the bottom shelf because of the difference in gravitational potential energy. An object's mass increases slightly when it is hot because the kinetic energy of the vibrating atoms is higher.

Because c^2 is such a large number, a very tiny mass is equivalent to a large amount of energy. In the early days of nuclear physics, $E = mc^2$ revealed the enormous energy locked up inside an atom's nucleus by the *strong nuclear force* that holds the protons and neutrons together. It was this that alerted nuclear physicists just before World War II to the possibility of a nuclear bomb. The energy released by the nuclear bomb dropped on Hiroshima at the end of that war (smallish by modern standards) resulted from a reduction in relativistic mass of about 0.7 g (slightly less than the mass of a standard wire paperclip).

Worked example

QUESTION

When free protons and neutrons become bound together to form a nucleus, the reduction in nuclear potential energy (binding energy) is released, normally in the form of gamma rays. Relativity says this loss in energy is reflected in a decrease in mass of the resulting atom.

PHYSICS PHILE

EVIL TWINS

The most extreme mass–energy conversion involves antimatter. For every kind of matter particle there is an equivalent antimatter particle, an 'evil twin', bearing properties (such as charge) of opposite sign. Particles and their antiparticles have the same rest mass. When a particle meets its antiparticle, they mutually annihilate—all their opposing properties cancel, leaving only their mass-energy, which is usually released in the form of two gamma-ray photons. Matter–antimatter annihilation has been suggested (speculatively) as a possible propellant for powering future interstellar spacecraft.

Discuss the implications of mass increase, time dilation and length contraction for space travel.

PHYSICS PHILE

EXPLODING A MYTH

It is commonly believed (wrongly) that Einstein was involved in the US nuclear bomb project. Perhaps this is because, during World War II, the nuclear physicists Leo Szilard, Eugene Wigner and Edward Teller, knowing such a bomb was possible and worried the Nazis might build one, wrote a letter to President Roosevelt suggesting the US beat them to it. They asked their friend Einstein to sign it because, being the most well-known scientist at the time, he would be taken seriously. Apart from that, Einstein did two days' work on the theory behind uranium enrichment.

Calculate how much energy is released when free protons, neutrons and electrons combine to form 4.00 g of helium-4 atoms (2 protons + 2 neutrons + 2 electrons). At room temperature and pressure, each 4 g of helium gas is about 25 L, roughly the volume of an inflatable beach ball.

Data: Mass of proton $m_p = 1.672622 \times 10^{-27}$ kg

Mass of neutron $m_n = 1.674927 \times 10^{-27}$ kg

Mass of electron $m_e = 9.11 \times 10^{-31}$ kg

Mass of helium atom $m_{He} = 6.646476 \times 10^{-27}$ kg

$c^2 = 8.9876 \times 10^{16}\ \text{m}^2\ \text{s}^{-2}$

SOLUTION

Total mass of the parts:

$$m_T = 2(m_p + m_n + m_e) = 2(1.672622 + 1.674927 + 0.000911) \times 10^{-27}\ \text{kg}$$
$$= 6.69692 \times 10^{-27}\ \text{kg}$$

Reduction in mass:

$$\Delta m = m_T - m_{He} = (6.69692 - 6.646476) \times 10^{-27}\ \text{kg}$$
$$= 5.0444 \times 10^{-29}\ \text{kg}$$

Binding energy per He atom:

$$\Delta E = \Delta mc^2 = 5.0444 \times 10^{-29}\ \text{kg} \times 8.9876 \times 10^{16}\ \text{m}^2\ \text{s}^{-2}$$
$$= 4.5337 \times 10^{-12}\ \text{J}$$

Binding energy for 4.00 g (0.004 kg):

$$\frac{4.5337 \times 10^{-12}\ \text{J}}{m_{He}} \times 0.004\ \text{kg} = 2.73 \times 10^{12}\ \text{J}$$

This much energy would be released by the explosion of more than 600 tonnes of TNT.

Some physicists dislike the definition of relativistic mass m_v of a moving object and prefer to talk only about the energy of an object (and its rest mass m_0). There are problems with the definition, including the fact that relativistic mass doesn't behave like a scalar, because it can be different along different directions.

CHECKPOINT 3.4

1 **Discuss** why, if Lorentz and Fitzgerald came up with the correct formula for length contraction, Einstein gets the credit for explaining relativistic length contraction.
2 Write the formula for length contraction. Would a ruler moving lengthwise relative to you appear shorter or longer?
3 **Define** the term *proper length*.
4 To what limit does observed length of a moving object tend as speed approaches *c*?
5 Write the formula for relativistic mass. Would a mass moving relative to you appear larger or smaller?
6 Use relativistic mass to **justify** the statement that the speed of light is the universal speed limit.
7 **Define** all the terms in the equation $E = mc^2$ and **explain** what the equation means.
8 **Explain** why an atom weighs less than the sum of its parts.

CHAPTER 3

This is a starting point to get you thinking about the mandatory practical experiences outlined in the syllabus. For detailed instructions and advice, use *in2 Physics @ HSC Activity Manual*.

ACTIVITY 3.1: FACT OR FICTION: INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

Perform an investigation to help distinguish between non-inertial and inertial frames of reference.

Perform an investigation that allows you to distinguish between inertial and non-inertial frames of reference.

Equipment: protractor, string, mass (50 g), tape, cardboard, chair on wheels or skateboard.

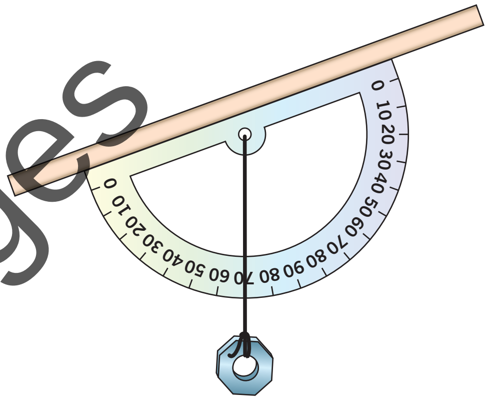

Figure 3.5.1 An accelerometer

Discussion questions

1. The principle method for detecting a non-inertial frame is measurement of acceleration. **Describe** an example of a non-inertial frame in which a typical accelerometer would *not* appear to measure an acceleration or detect extra fictitious forces.
2. Is there a test that can be performed within a frame of reference to tell if the effect measured by the accelerometer is the result of acceleration of the frame or due to an actual additional force?

ACTIVITY 3.2: INTERPRETING THE MICHELSON–MORLEY EXPERIMENT RESULTS

Gather and process information to interpret the results of the Michelson–Morley experiment.

Use simulations to gather data from the Michelson–Morley experiment. You will gather data as though there is and is not an aether, and then interpret the results.

There are many Michelson–Morley experiment simulations available. Two web-based examples are given on the companion website.

Discussion questions

1. **Describe** what Michelson and Morley were expecting to observe if aether were present.
2. Using the data you have gathered, **explain** how your observations support or refute the existence of the aether.
3. **Recall** the interpretation put forward by Michelson and Morley.
4. **Discuss** the importance of this experiment.

Extension

1. Research the history of how long the belief in aether persisted in some physicists after the publication of special relativity in 1905.
2. Read the following paper, which contains a thorough review of the history of the Michelson–Morley experiment, including historical letters to and from several researchers:
   Shankland, R S, 1964, 'Michelson–Morley Experiment', *American Journal of Physics*, vol. 32, p 16.

Chapter summary

- Inertial reference frames are those that do not accelerate.
- Principle of relativity: The laws of mechanics are the same in all inertial reference frames. Einstein extended it to all laws of physics (first postulate of relativity).
- When judged within a non-inertial frame, fictitious forces are perceived.
- Maxwell's equations for electromagnetism predicted only a single possible speed for light, which was assumed to be relative to a hypothetical medium called aether.
- Michelson and Morley failed to detect changes in speed due to aether wind, using an interferometer. Fitzgerald and Lorentz made the ad hoc suggestion that things contract when moving relative to the aether, hiding the effect of the changing relative speed of light.
- Einstein and others argued that aether was not required by Maxwell's equations and was inconsistent with the principle of relativity.
- Second postulate of relativity: The speed of light is constant to all observers.
- The speed of light is the fastest possible speed.
- The finite speed of light means different observers disagree on the simultaneity and order of events. Only events at the same time and place are agreed by all observers to be simultaneous.
- Lorentz factor: $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$
- Proper time t_0 is a time interval measured on a clock stationary in the observer's frame.
- Proper length l_0 is the length of an object stationary in the observer's frame.
- Proper or rest mass m_0 is the mass of an object stationary in the observer's frame.
- Time dilation: $t_v = \dfrac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

  Clocks (and all time-dependent phenomena) evolve in time more slowly if they are moving relative to the observer's frame.
- Length contraction: $l_v = l_0\sqrt{1-\dfrac{v^2}{c^2}}$

  Length l_v of an object moving relative to the observer's frame contracts in the direction of motion.
- Relativistic mass: $m_v = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

  Mass of an object m_v moving relative to the observer's frame increases.
- Two observers in separate inertial frames will agree on their relative speed v.
- However, both observers will judge the *other* observer to be moving and, hence, subject to time dilation, length contraction and relativistic mass increase. They disagree, but both are correct because these three quantities are relative. Only when two observers are in the same frame will they agree on these.
- Mass and energy are equivalent: $E = mc^2$. A small mass is equivalent to a large energy.

Review questions

PHYSICALLY SPEAKING

Use the words below to complete the following paragraph:

Galileo, Newton, Einstein's, Maxwell, constancy, fictitious, change, non-inertial frames, length, observer, classical, value, invariant, mass, time, frames, speed, inertial, special

Inertial ____________ have ____________ status in ____________ mechanics. ____________'s laws apply in these frames. If one performs measurements in ____________, then ____________ forces might be perceived. Classical mechanics and ____________ relativity both agree that physical laws are ____________ in ____________ frames. However, they disagree on the ____________ of the speed of light. According to ____________'s equations, the ____________ of the speed of light does not ____________ between frames, so light doesn't obey the transformation formula of ____________. Because of this, measurements of ____________, ____________ and ____________ within a reference frame moving relative to the ____________, will depend on the ____________ of that frame.

REVIEWING

1 You have a priceless Elvis Presley doll hanging from your rear-vision mirror at a constant angle from vertical. Elvis's feet lean towards the front of the car. Are you driving:

A forwards at uniform speed?
B backwards at uniform speed?
C forwards but accelerating?
D forwards but decelerating?

2 In a car that is cornering, is the centripetal force exerted on you by the seat belt fictitious? *Centrifugal force* normally refers to the fictitious force you feel pushing you outwards when you steer a car. Some people have suggested re-defining centrifugal force as the outward reaction force you exert on the seat belt in response to the centripetal force it exerts on you. Re-defined in this way, is centrifugal force still fictitious? **Justify** your answers.

3 At the end of the 19th century, no-one was able to travel at close to the speed of light, and clocks, rulers and mass balances weren't sensitive enough to measure relativistic changes. So why did the problems with classical physics start to become obvious then?

4 **Explain** why interferometry is an extremely sensitive method for measuring short differences in time or length.

5 **Explain** why Michelson and Morley performed their experiment at different times of the day and year.

6 If we were an entire civilisation of blind people relying on sound instead of light to decide the simultaneity of events, would our equations for relativistic length, time and mass contain $c = 340\ \text{m s}^{-1}$ (the speed of sound in air) instead? What's so special about the speed of light? **Discuss**.

7 In Figure 3.3.2b, the dimensions of the light path have been drawn correctly. However, for simplicity, two aspects of the train's appearance to observer v have been left out. **Describe** two changes that would need to be made to Figure 3.3.2b to represent these effects more correctly.

8 Suppose our relativistic twins Bill and Phil both got into spacecraft, went off in opposite directions and took journeys at relativistic speeds that were mirror images (judged from Earth). **Predict** and **explain**:

a how their apparent ages will compare when they come back home
b how their apparent ages will be judged by stay-at-home earthlings.

S Solve problems and analyse information using:

$E = mc^2$

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

9 Prunella and Renfrew, two observers in inertial frames moving relative to each other, will always agree on their relative speed *v*. A third observer, Thor, standing between them, sees them both coming towards him from opposite directions, at equal speeds. Is it correct to say that relative to Thor, Prunella and Renfrew are both moving at a speed of $v/2$?

10 A stretch-limo drove into a small garage at near light speed. The garage attendant slammed the garage door behind the car. For a brief time the attendant saw that the relativistically shortened limo was completely contained between the closed garage door and the rear garage wall. A short time later, the still-moving car smashed through the back wall. As far as the driver was concerned, the garage was shortened and the limo was too long for the garage so the limo was *never* contained between a closed door and the intact back wall. Reconcile the two differing accounts of what happened. (Hint: See section 3.3.)

11 Show that mc^2 has the units and dimensions of energy.

12 In a perfectly inelastic collision, two colliding objects stick together. In a symmetrical inelastic collision between two identical objects, the final speed is zero in the frame of their centre of mass. Given that mass-energy is conserved in an inertial frame, is the mass of the system the same as before the collision? **Explain**. (Hint: What happens to kinetic energy in an inelastic collision?)

SOLVING PROBLEMS

13 Depending on your answer to Question **1**, **calculate** the magnitude of your speed or acceleration if the Elvis Presley doll hangs at a constant angle of 10° from vertical.

14 The caption for Figure 3.2.3b states that increasing the length of the arms would increase sensitivity to changes in the speed of light. **Justify** this, using the equations given in that section.

15 Supposing the aether hypothesis were correct, show that (in agreement with Fitzgerald and Lorentz's suggestion) if the length *L* of the interferometer arm parallel to the aether wind shrinks to

$$L' = L\sqrt{1 - v^2/c^2}$$

then the difference $t_2 - t_1$ between the times of flight for the two arms would be zero. Use the equations given for t_2 and t_1 in section 3.2.

16 In the worked example of your trip to Proxima Centauri (Figure 3.4.4), one member of the crew had a mass 80 kg at launch. Assuming his normal diet and physiology were maintained, what would you expect his mass to be during the trip:

a as measured on the spaceship?

b as judged from Earth?

17 Your rival in the space race plans a trip to Alpha Centauri, which is slightly further away (4.37 ly). She wants to do the trip in 3.5 years (one-way) as judged by her own on-board clock.

a What speed (as a fraction of *c*) does she need to maintain?

b How long does the trip take as judged from Earth?

18 **Calculate** the total energy in the two gamma ray photons produced when an electron meets a positron (an anti-electron) ($m_e = 9.11 \times 10^{-31}$ kg).

19 For subatomic particles, a more conveniently sized (non-SI) unit of energy is the *electron volt* (eV). The conversion is $E(\text{eV}) = E(\text{J})/e$ where $e = 1.60 \times 10^{-19}$ C, the charge on an electron. A mega-electron volt (MeV) is 10^6 eV.

For the worked example on page 71, show that the kinetic energy of the electron in the medical linac beam is 6.4 MeV ($m_e = 9.11 \times 10^{-31}$ kg). What is the *total* energy of that electron?

20 Estimate the total energy (in joules) released by the Hiroshima bomb ($\Delta m_0 = 0.7$ g).

Solve problems and analyse information using:

$E = mc^2$

$$l_v = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

21 In their rest frame, muons have a mean lifetime of 2.2×10^{-6} s. However, measurements (at various altitudes) of muons produced by cosmic rays indicate that, on average, they travel 6.00×10^3 m from where they are produced in the upper atmosphere before decaying. **Calculate** their average speed (as a fraction of *c*).

22 Show that if the speed of light were infinite, the following equations would revert to their classical form.

a $l_v = l_0\sqrt{1 - v^2/c^2}$

b $t_v = \dfrac{t_0}{\sqrt{1 - v^2/c^2}}$

c $m_v = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$

d $\mathbf{v}_{(\text{A rel. to B})} = \dfrac{\mathbf{v}_A - \mathbf{v}_B}{1 - \mathbf{v}_A \mathbf{v}_B / c^2}$

23 Research the history of relativity and list up to five historically important experimental confirmations of its predictions. Make a timeline of the events. Note that some experiments may pre-date relativity. For example, in 1901 W Kaufmann measured the increase in an electron's mass as its speed increased. If possible, identify whether such examples came to Einstein's attention before he formulated his theory.

Analyse information to discuss the relationship between theory and the evidence supporting it, using Einstein's predictions based on relativity that were made many years before evidence was available to support it.

PHYSICS FOCUS

CAN'T MEASURE THE SPEED OF LIGHT

The French metric system, which evolved into the *Système International d'Unités* or SI units, was originally based on 'artefact' standards. The standard metre bar and kilogram were real objects (or artefacts) in Paris. Artefacts can degrade or be damaged, and making copies for standards labs is expensive, slow and unreliable. Artefact standards are now being replaced by fundamental physical property standards. One second is now defined as a certain number of periods of oscillation of a very stable light frequency in the spectrum of cesium-133 (in atomic clocks).

Measurement standards often involve sensitive interferometry. The metre was changed in 1960 from the original bar to a certain number of wavelengths (measured interferometrically) of a colour from the krypton-86 spectrum.

Being invariant, the speed of light is very useful for standards. Interferometric measurements of the speed of light became so precise that the weakest link was the experimental difficulty in reproducing the krypton-86 standard metre. So in 1983 the speed of light was fixed by definition at exactly 299 792 458 m s^{-1} and the standard metre was redefined as the distance travelled by light in 1/299 792 458 of a second. Now, any lab with an interferometer and an atomic clock can produce its own standard metre.

Since 1983, by definition, the speed of light can no longer be measured. Traditional procedures for measuring the speed of light should now be called 'measuring the length of a metre'.

The last artefact standard, the platinum–iridium kilogram in Paris, appears to be changing mass slightly. The Avogadro project at Australia's CSIRO is trying to develop a replacement for it with a procedure for making and testing (almost) perfect spheres of silicon that could be made in standards labs around the world without the need to copy the original sphere directly. The spheres are measured using interferometry, with the best result so far being an overall distortion from sphericity of 30 nm and an average smoothness of 0.3 nm.

S PFA 2. The nature and practice of physics

S PFA 3. Applications and uses of physics

S PFA 5. Current issues, research and developments in physics

S Discuss the concept that length standards are defined in terms of time in contrast to the original metre standard.

1. **Explain** why standards based on fundamental physics properties are preferable to artefacts.
2. **Justify** (in light of relativity) the statement that the speed of light is an especially good property on which to base a measurement standard.
3. The 1960 metre standard was based on light from krypton-86. **Explain** why it needed to specify the light source and why the new metre standard doesn't.
4. Given that the value of the speed of light is now arbitrarily fixed, **discuss** why they didn't just make the speed of light a nice round number such as $3.000\,000\,00 \times 10^{8}$ m s^{-1}.
5. A single atomic layer of silicon is approximately 5.4×10^{-10} m thick. For the best silicon sphere in the Avogadro project, to approximately how many atomic layers does the reported distortion from sphericity and average smoothness correspond?

Figure 3.5.2 One of CSIRO's accurate silicon spheres

MODULE 1

Review

The review contains questions in a similar style and proportion to the HSC Physics examination. Marks are allocated to each question up to a total of 30 marks. It should take you approximately 54 minutes to complete this review.

Multiple choice

(1 mark each)

1 Ignoring air resistance, all projectiles fired horizontally from the same height above horizontal ground will have the same:

A horizontal velocity.
B time of flight.
C range.
D final speed.

2 Which of the following orbits has a two-body mechanical energy greater than zero?

A Geostationary
B Elliptical
C Parabolic
D Non-returning comet

3 You have just rounded the top of a curve on a roller-coaster. The g-force meter you are carrying reads exactly zero. Which one of the following is true?

A Your weight is the centripetal force.
B Your weight is zero.
C Your weight is equal and opposite to the normal force exerted on you by the seat.
D Your weight is equal and opposite to the tension in your body.

4 The Michelson–Morley experiment demonstrated that:

A the aether wind was undetectable.
B waves do not require a medium.
C one arm of the interferometer contracted in response to the aether wind.
D aether is trapped by mountains and valleys and dragged along with the Earth.

5 Observer A on the ground, watches a train (containing observer B) rush past at speed *v*. Both make measurements of things in each other's frame of reference. From the following list of statements, choose the statement they *disagree* on.

A The other observer's frame of reference is moving with speed *v*.
B The apparent length of my own metre ruler is longer than the apparent length of other observer's metre ruler.
C Observer B's watch appears to run slower than observer A's watch.
D The height of the train carriage ceiling is 2.2 m above the carriage floor.

Short response

6 The escape velocity from the Earth's surface, based on Newton's original concept, is 11.2 km s^{-1}. Briefly explain two ways in which this number is not quite applicable to real Earth-surface launches. (2 marks)

7 Calculate the potential energy of a 2500 kg satellite in a geostationary orbit around the Earth. Assume a sidereal day is 23 h 56 min 4 s. (3 marks)

8 In their rest frame, charged pions have a mean lifetime of 2.60×10^{-8} s. A particular beam of charged pions travel an average distance of 30 m before decaying. Calculate their speed (as a fraction of the speed of light). (4 marks)

9 Explain why if you are in a circular orbit and you briefly retro-fire your engines to slow down, you move to a faster orbit. (3 marks)

10 A 9000 kg helicopter is parked at the equator and then later near the North Pole. Assuming the two locations are chosen so that the gravitational potential energy is the same at the two spots, estimate the difference in relativistic mass at the two locations. At which location will the mass be larger? (Hint: Velocity is low, so use the classical expression for kinetic energy.) (3 marks)

Extended response

11 Critically discuss the following proposition: 'The Michelson–Morley experiment was an embarrassment for physics because, despite a large effort, it failed to find what it was looking for and so it should be relegated to the dustbin of physics history.' (5 marks)

12 The following formula relates the length of a pendulum (L) to the period of its swing (T).

$$T = 2\pi\sqrt{\frac{L}{g}}$$

During your studies in physics you carried out an experiment to determine acceleration due to gravity. Describe and explain a method you would use to perform this measurement. (5 marks)