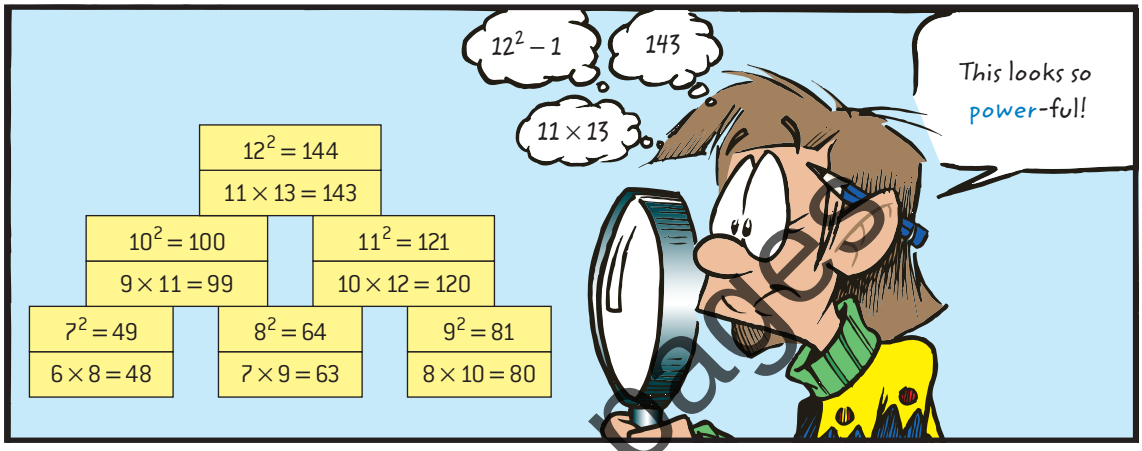
NUMBER AND INDICES

3



Contents

Syllabus references

(See pages x–xiii for details.)

Number and Algebra

Selections from ***Indices*** [Stage 4]

- Investigate index notation and represent whole numbers as products of powers of prime numbers. (ACMNA149)
- Investigate and use square roots of perfect square numbers. (ACMNA150)

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

3:01 Index notation

PREP QUIZ 3:01

Find the value of:

1 $3 \times 3 \times 3$ **2** 5×5 **3** $2 \times 2 \times 2 \times 2 \times 2$
4 $5 \times 5 \times 5$ **5** 10×10 **6** $10 \times 10 \times 10$
7 $10 \times 10 \times 10 \times 10$ **8** $10 \times 10 \times 10 \times 10 \times 10$ **9** $7 \times 10 \times 10$
10 $4 \times 10 \times 10 \times 10$

We use a **power** when a number is in a product more than once.

$5^2 = 5 \times 5$
This is five squared, or five to the power of two.
$10^3 = 10 \times 10 \times 10$
This is ten cubed, or ten to the power of three.

Note:
Another name for **power** is **index**.

WORKED EXAMPLES

1 Write 5^3 in expanded form and write its basic numeral.

2 Write $10 \times 10 \times 10 \times 10$ as a power of ten.

3 What is the basic numeral of 7×10^3?

5^3 is $5 \times 5 \times 5$ written using **index notation**.

Solutions

1 5^3
$= 5 \times 5 \times 5$
$= 25 \times 5$
$= 125$

2 $10 \times 10 \times 10 \times 10$
$= 10^4$

3 7×10^3
$= 7 \times (10 \times 10 \times 10)$
$= 7 \times 1000$
$= 7000$

Exercise 3:01

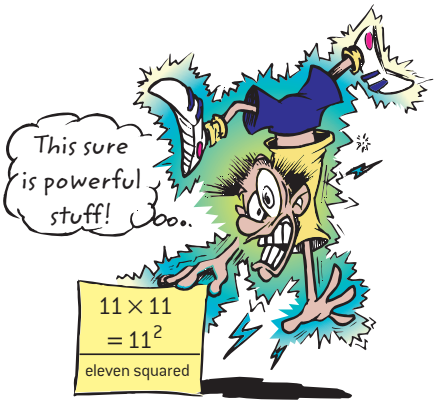
P Foundation worksheet 3:01 Powers of numbers

1 Write each in expanded form and as a basic numeral.

a 4^2 b 10^2 c 2^2
d 1^2 e 7^2 f 9^2
g 2^3 h 5^4 i 3^3
j 10^3 k 10^4 l 10^5

2 Rewrite using index notation.

a 7×7 b $8 \times 8 \times 8$
c $10 \times 10 \times 10 \times 10$ d $10 \times 10 \times 10$
e $4 \times 4 \times 4 \times 4 \times 4 \times 4$ f $2 \times 2 \times 2 \times 2$
g $10 \times 10 \times 10 \times 10 \times 10$



3 Write the basic numeral for each.

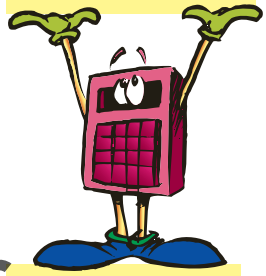
a 6×10^1	**b** 3×10^2	**c** 5×10^3
d 2×10^4	**e** 7×10^3	**f** 1×10^2
g 4×10^3	**h** 9×10^2	**i** 8×10^4
j $3^2 \times 10^3$	**k** $2^3 \times 10^4$	**l** $7^2 \times 10^5$

A basic numeral is the simplest answer.

4 Rewrite each expression using index notation.

a $2 \times 2 \times 2 \times 3 \times 3$	**b** $5 \times 5 \times 2 \times 2 \times 2$
c $4 \times 4 \times 3 \times 3 \times 3 \times 3$	**d** $6 \times 6 \times 6 \times 7 \times 7$
e $5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3$	**f** $8 \times 8 \times 8 \times 10 \times 10$
g $2 \times 2 \times 3 \times 2 \times 3 \times 3$	**h** $5 \times 6 \times 5 \times 5 \times 6$
i $4 \times 4 \times 5 \times 4 \times 4$	**j** $3 \times 7 \times 3 \times 7 \times 7 \times 3$

$3 \times 3 \times 3 \times 3 \times 5 \times 5$
$= 3^4 \times 5^2$



5 Rewrite each in expanded form and then as a basic numeral.

a $2^2 \times 3^2$	**b** $5^2 \times 3^2$
c $4^2 \times 2^3$	**d** $3^3 \times 2^2$
e $3^2 \times 2^5$	**f** $5^3 \times 2^3$
g $4^3 \times 3^2 \times 2^2$	**h** $2^3 \times 3^2 \times 4^1$

$2^3 \times 5^2$
$= 2 \times 2 \times 2 \times 5 \times 5$
$= 8 \times 25$
$= 200$

6 Rewrite each in expanded form, then as one term with only one power.

a $2^2 \times 2^3$	**b** $3^4 \times 3^3$
c $5^3 \times 5^5$	**d** $4^3 \times 4^2 \times 4^1$

Do you see any pattern or rule?

7 Write each expression as a basic numeral, then express this numeral as a power of a single number.

a $2^2 \times 3^2$	**b** $2^2 \times 5^2$
c $4^2 \times 2^2$	**d** $5^3 \times 2^3$

Do you see any pattern or rule?

$20^2 = (2 \times 10)^2$
$= 2^2 \times 10^2$

8 Evaluate each expression, remembering the rules for order of operations.

a $2 \times 2 + 3 \times 3$	**b** $2^2 + 3^2$	**c** $3^2 - 2^2$
d $2 \times 2 \times 2 + 3 \times 3 \times 3$	**e** $2^3 + 3^3$	**f** $3^3 - 2^3$
g $2 \times 3^2 + 3 \times 2^3$	**h** $4 \times 2^3 + 5^2$	**i** $3^3 - 5 \times 2^2$
j $5 \times 2^2 + 2 \times 6^2$	**k** $4^2 + 2^2 \times 5^2$	**l** $5 \times 2^4 - 3^2 \times 2^3$
m $5^3 \times 2^2 - 2^4 \times 6$	**n** $4^3 \div 2^2 \times 5$	**o** $5^2 \times 2^3 - 10^2 \div 2^2$

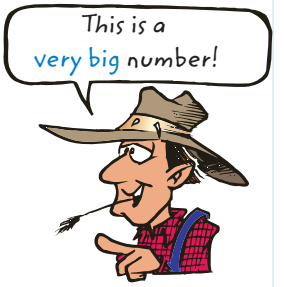
9 Use a calculator to find the value of the following. (See the examples at right.)

a 2^5	**b** 3^4	**c** 4^4
d 2^9	**e** 3^7	**f** 5^6
g 6^4	**h** 7^5	**i** 9^4
j 12^3	**k** 20^4	**l** 18^5
m $5^3 + 2^6$	**n** $4^4 - 3^5$	
o $3^6 + 7^3$	**p** $10^3 - 5^4$	
q $8^3 + 2^5 \times 6$	**r** $6^5 \div 3^4 + 4$	
s $5^5 - 2^6 \times 4^2$	**t** $4^7 \div 8^3 \times 5^4$	
u $9^3 \times 2^{10} - 7^4 \times 6^3$	**v** $6^7 - 5^6 \times 4^8 \div 8^4$	

To find 3^6 using a calculator:
Enter: 3 $x^{\blacksquare}$ 6 =
Answer: 729

NOTE
For $2^7 + 5^4$:
Enter: 2 $x^{\blacksquare}$ 7) +
5 $x^{\blacksquare}$ 4 =
Answer: 753

CHALLENGE 3:01 NOW THAT'S A GOOGLE



Almost everyone is familiar with the word 'google' from the internet. However, this brand name originally comes from the word **googol** (with different spelling), which was first used in 1938 to refer to the very large number 10^{100}.

So 1 googol = 10^{100}.
This number is 1 followed by 100 zeros:

10 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

However, a **googolplex** is even bigger—it is a much bigger number than a googol.

- Find out how big a googolplex is.
- How long would it take you to write a googol, i.e. 1 followed by 100 zeros?
- How long would it take you to write a googolplex?

3:02 Expanded notation

PREP QUIZ 3:02

If $10^2 = 10 \times 10$, find the simplest numeral for:

1 10^2 **2** 5×10^2 **3** 8×10^2

If $10^3 = 10 \times 10 \times 10$, find the simplest numeral for:

4 10^3 **5** 2×10^3 **6** 9×10^3

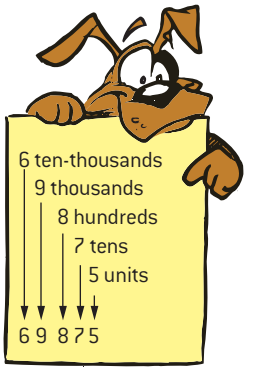
Write in simplest form:

7 $(6 \times 10) + 3$ **8** $(2 \times 10^2) + (5 \times 10) + 1$

9 $(7 \times 10^3) + (9 \times 10^2) + (3 \times 10) + 5$ **10** $(8 \times 10^3) + (1 \times 10^2) + (3 \times 10) + 2$

$$69\,875 = (6 \times 10\,000) + (9 \times 1000) + (8 \times 100) + (7 \times 10) + (5 \times 1)$$
$$= (6 \times 10^4) + (9 \times 10^3) + (8 \times 10^2) + (7 \times 10) + (5 \times 1)$$

Ten-thousands 10 000	Thousands 1000	Hundreds 100	Tens 10	Units 1
$10 \times 10 \times 10 \times 10 = 10^4$	$10 \times 10 \times 10 = 10^3$	$10 \times 10 = 10^2$	$10 = 10^1$	$1 = 1$
6	9	8	7	5



WORKED EXAMPLES

1 Write 6 millions + 9 thousands + 2 hundreds + 5 tens as a simple numeral.

2 Write $(5 \times 10^4) + (7 \times 10^2) + (2 \times 10^1) + (1 \times 1)$ as a numeral in its simplest form.

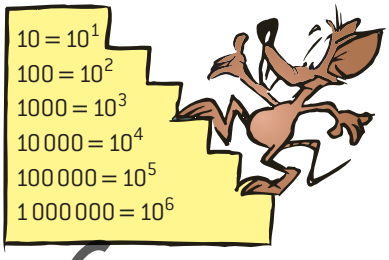
3 Write 932 014 in expanded notation.

Solutions

1 6 millions + 9 thousands + 2 hundreds + 5 tens

Column values						
1 000 000	100 000	10 000	1 000	100	10	1
6	0	0	9	2	5	0

Zeros act as place holders, allowing the other digits to be in their correct columns.



2 $(5 \times 10\,000) + (0 \times 1000) + (7 \times 100) + (2 \times 10) + (1 \times 1)$
$= 50\,721$

3 $932\,014$
$= (9 \times 100\,000) + (3 \times 10\,000) + (2 \times 1000) + (0 \times 100) + (1 \times 10) + (4 \times 1)$
$= (9 \times 10^5) + (3 \times 10^4) + (2 \times 10^3) + (0 \times 10^2) + (1 \times 10^1) + (4 \times 1)$

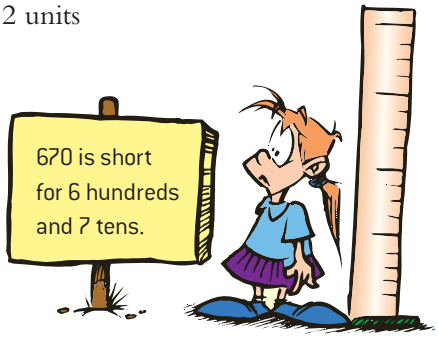
Exercise 3:02

1 Write each as a simple numeral (a basic numeral).

- **a** 6 thousands + 4 hundreds + 5 tens + 9 units
- **b** 2 ten-thousands + 8 thousands + 6 hundreds + 3 tens
- **c** 9 hundred-thousands + 8 hundreds + 7 tens + 4 units
- **d** 4 millions + 3 hundred-thousands + 8 ten-thousands
- **e** 7 ten-thousands + 4 thousands + 5 hundreds + 8 tens + 6 units
- **f** 1 million + 1 hundred-thousand + 1 thousand + 1 ten
- **g** 5 millions + 6 ten-thousands + 8 hundreds + 4 tens + 2 units
- **h** 4 ten-thousands + 8 thousands + 3 hundreds + 9 tens

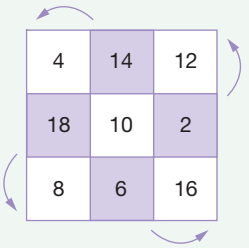
2 Write these numerals in simplest form.

- **a** $(8 \times 1000) + (5 \times 100) + (3 \times 10) + (9 \times 1)$
- **b** $(7 \times 100) + (3 \times 10) + (8 \times 1)$
- **c** $(7 \times 1000) + (3 \times 100) + (0 \times 10) + (4 \times 1)$
- **d** $(9 \times 1000) + (0 \times 100) + (6 \times 10) + (7 \times 1)$
- **e** $(9 \times 1000) + (0 \times 100) + (0 \times 10) + (3 \times 1)$
- **f** $(8 \times 1000) + (2 \times 100) + (1 \times 10) + (0 \times 1)$



Step 4

Rotate the square anticlockwise to give it the appearance below.



Activity

1 Use this method to make magic squares with the following numbers:
   - **a** 3, 4, 5, 6, 7, 8, 9, 10 and 11
   - **b** 5, 10, 15, 20, 25, 30, 35, 40 and 45
   - **c** 36, 33, 30, 27, 24, 21, 18, 15 and 12

2 What is the sum of a column in the magic squares of **1** (above)?

3 Use the pattern 1, 2, 4, 8, 16, 32, 64, 128, 256 to make a magic square using this method. What is the product of numbers in each row, column and diagonal?

MATHS TERMS 3

basic numeral
- the simplest way to write a number, e.g. The basic numeral for $(4 + 8) \times 2$ is 24.

composite number
- a number that has more than two factors, e.g. 9 is composite because it has three factors: 1, 3 and 9

cube (number)
- the answer when a whole number is a product of itself 3 times, e.g. $5^3 = 5 \times 5 \times 5 = 125$

cube root ($\sqrt[3]{\ }$)
- to find the cube root of a number, e.g. 8, find the number that needs to be cubed to give 8: $2^3 = 8$ so $\sqrt[3]{8} = 2$

expanded notation
- a way of writing a number as the sum of its parts, e.g. $612 = (6 \times 100) + (1 \times 10) + (2 \times 1)$

factor
- a factor of a counting number divides it exactly, e.g. The factors of 6 are 1, 2, 3 and 6.
- common factor: a number that is a factor of all numbers being considered, e.g. 7 is a common factor of 14, 21 and 70.
- highest common factor (HCF): the largest of the common factors, e.g. 18 and 24 have common factors 2, 3 and 6, but the highest common factor is 6.

index (plural: indices)
- a number indicating how many of a base number need to be multiplied together, e.g. for 5^3, the index is 3

multiple
- a multiple of a counting number is found by multiplying it by another counting number, e.g. The multiples of 5 are 5, 10, 15, 20, …
- common multiple: a number that is a multiple of all numbers being considered, e.g. 50 is a common multiple of 2 and 5.
- lowest common multiple (LCM): the smallest of the common multiples, e.g. 10 is the LCM of 2 and 5. 20 is the LCM of 2, 5 and 10.

power
- another word for *index*

prime number
- a counting number that has exactly two factors, itself and 1, e.g. 17, 31, 2

square number
- the result of multiplying a counting number by itself, e.g. 1, 16, 25

square root ($\sqrt{\ }$)
- the square root of a number, e.g. What is the number that must be squared to give 64? $8^2 = 64$ so $\sqrt{64} = 8$

DIAGNOSTIC TEST 3 NUMBER AND INDICES

Each section of the test has similar items that test a certain type of example.
Errors in more than one item will identify an area of weakness.
Each weakness should be treated by going back to the section listed.

1 Write each as a power. 3:01
a $8 \times 8 \times 8$ **b** $3 \times 3 \times 3 \times 3 \times 3$ **c** $2 \times 2 \times 2 \times 2$

2 Write the basic numeral for each. 3:01
a 8×10^2 **b** 2×10^4 **c** 7×10^6

3 Write each as a basic numeral. 3:02
a $(7 \times 10^3) + (2 \times 10^2) + (2 \times 10^1) + (8 \times 1)$
b $(5 \times 10^3) + (0 \times 10^2) + (8 \times 10^1) + (0 \times 1)$

4 Write each in expanded form using powers of ten. 3:02
a 1824 **b** 4307 **c** 2 415 286

5 List all the factors of: 3:03
a 24 **b** 63 **c** 100

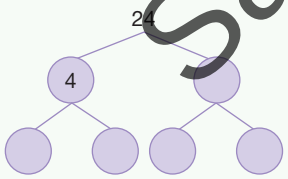
6 List the first five multiples of: 3:03
a 4 **b** 8 **c** 11

7 Find the highest common factor of: 3:03
a 36 and 48 **b** 60 and 75 **c** 70 and 98

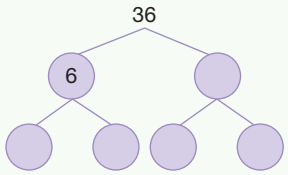
8 Find the lowest common multiple of: 3:03
a 12 and 9 **b** 6 and 8 **c** 20 and 14

9 **a** Write all prime numbers that are less than 10. 3:04
b Write all composite numbers that are less than 10.
c Which counting number is neither prime nor composite?

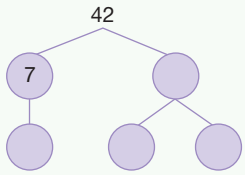
10 Complete these factor trees. 3:04
a



b



c



11 Find the highest common factor and the lowest common multiple of: 3:06
a $2 \times 2 \times 2 \times 3$ and $2 \times 2 \times 5$ **b** $3 \times 3 \times 3 \times 5 \times 7$ and $3 \times 5 \times 5 \times 7$
c $2 \times 3 \times 5 \times 7$ and $3 \times 3 \times 5$

Find the simplest answer for:

12 **a** $\sqrt{25}$ **b** $\sqrt{(7 \times 11) \times (7 \times 11)}$ **c** $\sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 3}$ 3:07

13 **a** $\sqrt[3]{27}$ **b** $\sqrt[3]{11 \times 11 \times 11}$ **c** $\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$ 3:07

ASSIGNMENT 3A Chapter review

1 Rewrite using index notation:
- **a** $5 \times 5 \times 5 \times 5$
- **b** $3 \times 3 \times 2 \times 2 \times 2$
- **c** $7 \times 4 \times 7 \times 4 \times 7$

2 **a** Write each as a basic numeral.
- **i** 4×10^3
- **ii** 5×10^4

b Hence, find the basic numeral for:
- **i** $(4 \times 10^3) + (5 \times 10^4)$
- **ii** $(4 \times 10^3) \times (5 \times 10^4)$

3 Rewrite as a basic numeral:
- **a** $(5 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (9 \times 1)$
- **b** $(8 \times 10^4) + (6 \times 10^2) + (3 \times 1)$

4 List, in order, the factors of:
- **a** 28
- **b** 48
- **c** 68

5 For each of the following, find the greatest multiple that is less than 100.
- **a** 3
- **b** 7
- **c** 15

6 'Prime number pairs' are pairs of prime numbers that differ only by 2, e.g. 11 and 13, or 41 and 43.
Find all the other prime number pairs between 10 and 100.

7 **a** By completing factor trees, find the prime factors of:
- **i** 56
- **ii** 84

b Find the HCF of 56 and 84.
c Find the LCM of 56 and 84.

8 Find the simplest answer for:
- **a** $\sqrt{121}$
- **b** $\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$
- **c** $\sqrt[3]{64}$
- **d** $\sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7}$

ASSIGNMENT 3B Working mathematically

1 A bookcase has six shelves, with 45 books on each shelf. How many books are in the bookcase?

2 A bus can hold 58 people seated and 35 standing. How many of these buses would be needed to take a school of 750 children and 30 teachers on a school excursion?

3 A grocer bought 50 kg of tomatoes for $30. She sold 20 kg at $1.20 per kg and the remainder, which were overripe, she sold for 80c per kg. How much did she get for selling the tomatoes? How much profit did she make?

4 In Rugby Union, a team scores 5 points for a try, 7 points for a converted try and 3 points for a penalty goal. In how many ways could a team score 22 points?

5 A school offers the following sports choices:
Summer: cricket, water polo, basketball, volleyball, tennis
Winter: football, hockey, soccer, squash

If one summer sport and one winter sport are selected, how many different combinations are available?

ASSIGNMENT 3C Cumulative revision

1 Write the basic numeral in each case. 1:06

a $12 \div (28 - 24)$ **b** $84 - 10 \times 8$ **c** $100 - [50 - (20 - 7)]$

d $23 + 5 - 2 + 3$ **e** $6 \times 5 + 6 \times 3$ **f** $3 + 7(0{\cdot}7 + 1{\cdot}3)$

2 We sell buckets of blocks. In each bucket there are 157 blocks. We sold 93 buckets of blocks on Monday and 7 on Tuesday. How many blocks did we sell on those two days altogether? 1:03

3 Given that $12 \times 1256 = 15\,072$, what is the value of: 1:07A

a 1256×12 **b** 13×1256 **c** 11×1256?

4 True or false? 1:07

a $2 \times 3186 \times 5 = 31\,860$ **b** $9 \times 888 = 8880 - 888$

c $555 \times 12 = (555 \times 6) \times 2$ **d** $18\,158 \div 20 = (18\,158 \div 2) \div 10$

e $4 \times 186 \times 25 = (4 \times 25) \times 186$ **f** $16 \times 100 - 1 = 16 \times 99$

5 Write as a basic numeral: 1:03

a 1846×1 **b** $0 \times 26\,040$ **c** $8145 + 0$

d $446 - 0$ **e** $0 \div 186$ **f** $4186 \div 1$

g $23 \times 11 - 22 \times 11$ **h** $31 \times 98 + 69 \times 98$ **i** $98 \times 37 + 2 \times 37$

6 True or false? 1:08

a $8 \times 3 \neq 3 \times 8$ **b** $563 < 560$ **c** $213 \times 0 \geq 0$

d $\sqrt{16} = 8$ **e** $16 \times 5 \leq 16 \times 4$ **f** $5 \times 816 > 0$

g $816 \div 1 \not< 816 \times 1$ **h** $999 \leqslant 100 - 0$ **i** $\sqrt[3]{1} = 1$

7 Find the answer to each calculation. 1:06

a $5 \times 4 + 3 \times 2$ **b** $5 + 4 \times 3 + 2$ **c** $5 \times (4 + 3) - 2$

d $(5 + 4) \div 3 + 2$ **e** $5 \times 4 \div (3 + 2)$ **f** $(5 - 4 + 3) \div 2$

8 **a** List the factors of 100. 3:03

b List the factors of 125.

c List all the common factors of 100 and 125.

9 **a** List the first 10 multiples of 8. 3:03

b List the first 10 multiples of 12.

c What is the lowest common multiple of 8 and 12?

10 From the set $\{2, 5, 7, 12, 15, 36, 41\}$, write: 3:04

a the prime numbers **b** the composite numbers.