## EQUATIONS



## Contents

9:01 Inverse operations
9:02 Solving equations Fun spot 9:02 Why did the tooth get dr
9:03 Equations with pronumerals on bothsides GeoGebra activity 9:03 Equations with pronumerals on both sides

Equations with grouping symbols GeoGebra activity 9:04 Equations with grouping symbols
9:05 Solving simple quadratic equations
9:06 Solving problems using equations
Maths terms, Diagnostic test, Assignments

## Syllabus references see page x for details.)

Number and Algebra
Selections from Equations [Stage 4]

- Solve simple linear equations. (ACMNA179)
- Solve linear equations using algebraic techniques and verify solutions by substitution. (ACMNA194)
- Solve simple quadratic equations.


## Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

In Year 7 you were introduced to equations. An equation is a number sentence in which one (or more) of the numbers is unknown and is represented by a pronumeral, usually a letter of the alphabet.

$$
\text { e.g. } x+5=12 \text { or } 10-3 a=4
$$

We solve an equation to find the value of the pronumeral.

Equations are sometimes called algebraic sentences.

To solve equations it is necessary to understand inverse operations.

## 2:01 Inverse operations

## PREP QUIZ 9:01

Write an algebraic expression for each of the following sentences.
1 Add 6 to $x$. 2 Multiply a by 8. 3 Divide $y$ by 7 . 4 Take 3 away from $p$.
5 Multiply $x$ by 3 and then add 5 .
6 Subtract 4 from $y$ and then multiply the result by 9 .
What is the opposite of:
7 adding 7
8 subtracting 8
10 dividing by 10 ?

We start with a pronumeral when building algebraic expressions.


- Arrow diagrams can be used to show how the expression has been made.
- The operation that has caused the change is written above the arrow.
- When solving equations, we need to reverse the steps to get back to the pronumeral. In other words, we need to perform the inverse operations.


Inverse means opposite.

$$
\begin{array}{ll}
2 & +5 \\
7 & \rightarrow \\
7 & -5 \\
\end{array}
$$

Adding 5 and subtracting 5 are inverse operations.

This process is called backtracking.

## WORKED EXAMPLES

1 Complete the following arrow diagrams by writing the correct operation above each arrow.
$a x \rightarrow x+6$
c $a \rightarrow a-4 \rightarrow 2(a-4)$
b


2 Use the inverse operations to get back to the pronumeral.
$a \underset{\rightarrow}{\rightarrow} \square$
c

c $a \rightarrow a+4 \rightarrow 2(a-4)$


## Solutions

1 a $x \rightarrow x+6$

$$
5 y+7 \xrightarrow{-7} \square \rightarrow 5
$$

d

b $y \xrightarrow{\times 5} 5 y \xrightarrow{+7} 5 y+7$
$\mathbf{d} \square \underset{\square}{-8} \div\left(\frac{a-8}{9}\right.$

2 a $x+6 \rightarrow \underset{\rightarrow}{\rightarrow}$
b $5 y+7 \rightarrow \stackrel{-7}{\rightarrow} 5 y+5$
c $2(a-4) \stackrel{\div 2}{\rightarrow} a-4 \rightarrow a$
d $\square$ $\stackrel{\times 9}{\rightarrow} a-8$ $\xrightarrow{+8} \square$

1 Complete the following arrow diagrams by writing the correct operation above each arrow.
a $\square$
b $m \rightarrow m-5$
c

d

e$y \rightarrow y+1 \rightarrow 3(y+1)$$\mathrm{f} p \rightarrow p-6 \rightarrow 9(p-6)$

$$
\mathrm{g} x \rightarrow x+1 \rightarrow \frac{x+1}{2}
$$

$$
\mathrm{h} \rightarrow t \rightarrow \frac{t-7}{10}
$$

$$
\mathrm{i} \square \rightarrow \frac{q}{2} \rightarrow \frac{q}{2}+3
$$



$$
\mathrm{k} \square \rightarrow \frac{m}{-6} \rightarrow \frac{m}{-6}-8
$$

$$
\stackrel{a}{\mathrm{~L}} \rightarrow-5 a \rightarrow 2-5 a
$$

2 Draw arrow diagrams to show hoy the following expressions were made from each pronumeral.
a 9 a

c $q-9$
d $\frac{x}{2}$
e $5 n+6$
i $\frac{x}{3}+2$
$3+\frac{n}{5}$
$n 8(x-1)$
g $4 p+3$
h $9 y-5$
k $\frac{x+7}{2}$
( $\frac{a-1}{4}$
m 5(n+2)

- $2(q+9)$
p $10(t-1)$
q $5-6 x$
r $-7 v+1$
s $\frac{7-a}{2}$
t $9-\frac{x}{6}$

3 Write the operations that must be performed on $x$ to produce the following expressions.
a $7 x$
b $x-7$
c $x+1$
d $\frac{x}{8}$
e $3 x+5$
f $7 x+2$
g $4 x-9$
h $6+4 x$
i $\frac{x}{5}-7$
j $10+\frac{x}{5}$
k $\frac{x-7}{3}$
( $\frac{x+1}{2}$
m 3( $x+7$ )
n $9(x-4)$

- $-7 x+1$
p $12-x$

4 Complete the following arrow diagrams.
a $\square$
b
$\square$
c

$\square$
d
$\square$
e

f

$\square$
g

$\square$
h $\square$i $\quad q \xrightarrow{-3}$

$\square$
j
$n \xrightarrow{\times(-2)} \square \xrightarrow{+6} \square$
$\mathrm{k} \quad b+10 \xrightarrow{-10}$ $\square$
$\mathrm{m} \stackrel{5 m}{\rightarrow} \stackrel{+}{\rightarrow}$ $\square$
l


- $2 x+3$

$\square$
q $6+7 t$

$\square$
n
$\stackrel{n}{4} \xrightarrow{\times 4} \times x$p $5 p-4$

$$
] \xrightarrow{+4}[
$$

r $\frac{m+5}{4}$$\stackrel{\times 4}{ }$
s $\frac{a}{3}-5$


5 Complete these arrow diagrams, showing how to bet back to the pronumeral.


6 What operations must be performed on these expressions to backtrack to $a$ ?
a $5 a$
b $a-4$
c $a+9$
d $\frac{a}{8}$
e $2 a+1$
f $7 a+5$
g $8 a-3$
h $6+5 a$
i $\frac{a}{2}-5$
j $12+\frac{a}{5}$
k $\frac{a-7}{10}$
し $\frac{a+1}{3}$
m $8(a+7)$
n 2(a-11)

- $-3 a+1$
p 4-a

7 What order of inverse operations must be performed to get back to the pronumeral?
a $9(2 a-5)$
b $5(8 y+7)$
c $3(2+9 q)$
d $\frac{3 x+5}{2}$
e $\frac{5 x}{3}-2$
f $3+\frac{4 n}{5}$
g $\frac{7+3 x}{2}$
h $\frac{9 a-1}{4}$
i $4(5-6 x)$
j $-7(2 v+1)$
k $\frac{7-3 a}{2}$
l $9-\frac{x}{6}$

## 2:02Solving equatione

## PREP QUIZ 9:02

Simplify each of the following.
$17 x \div 7$
$2 x+4-4$

What is the inverse of:
5 multiplying by 36 adding 7

$3 a-7+7$
$4 \frac{m}{5} \times 5$

7 subtracting 1
8 dividing by 5?
Complete these arrowing diagrams, puthing operations above each arrow.
$95 m+3 \rightarrow 5 m \rightarrow$

- Solving equations is ine balancing scales.
- With equations, we know that one side is equal to the other.
- The solution of the equation therefore is the value of the pronumeral that 'balances' the equation.

$x=3$ balances the scale so $x=3$ is the solution.
$a=6$ balances the scale
so $a=6$ is the solution.

$8 a-2 \rightarrow 8 a \rightarrow a$
10

$$
8 a-2 \rightarrow 8 a \rightarrow a
$$

- Often, solving an equation requires us to change the equation into a simpler one. To do this we add $(+)$, subtract $(-)$, multiply $(\times)$ or divide $(\div)$ both sides of the equation by the same number.
- Look at the solutions of these two equations, noting that both sides remain balanced because the same operation is done to both sides.

We need to perform operations that will leave only the pronumeral on one side of the equation.


## WORKED EXAMPLE 2

Solve each of these two-step equations.
a $2 a+5=7$
b $\frac{x+3}{5}=2$
$-5 \quad-5$
$2 a=2$
$\div 2 \quad \div 2$

$$
\begin{aligned}
x 5 & \times 5 \\
x+3 & =10 \\
-3 & -3 \\
\therefore x & =7
\end{aligned}
$$

c $\begin{aligned} \frac{p}{3}-6 & =15 \\ & +6\end{aligned}$
$\frac{p}{3}=21$
$\times 3 \times 3$
$\therefore p=63$
d $10-3 m=25$
$-10 \quad-10$
$-3 m=15$
$\div-3 \div-3$
$\therefore m=-5$

## Exercise 9:02

1 Solve each of these one-step equations.
a $x+3=12$
b $q+9=13$
d $9+n=15$
e $8+p=14$
g $x-7=2$
h $w-8=8$
j $5 a=20$
k $7 y=35$
m $\frac{x}{2}=5$
n $\frac{a}{4}=3$
$)_{0}^{1}$
CH ${ }^{3}$
$3=21$
$m+t=33$
$m-5=11$
l $4 y=28$

- $\frac{m}{6}=6$

2 Solve these one-step equations that involve negative integers.
a $x+3=1$
d $2+n=-5$
g $n-7=-2$
j $3 a=-12$
$\mathrm{m}-2 a=-8$
$\mathrm{n}-8 y=-24$
c $a+13=8$
f $7+y=0$
i $q-5=-5$
l $5 x=-55$

- $-6 x=-42$
p $\frac{x}{2}=-4$
s $x-(-7)=1$
q $\frac{w}{4}=-1$
r $\frac{v}{-3}=-5$
t $t-(-8)=10$
u $q-(-5)=-5$

3 Substitute the solution into each equation below and state whether the solution is correct or not.
a $x+17=39$
b $9+m=15$
c $q-13=16$
d $n-28=17$
$x=56$
$m=7$
$q=29$
$n=11$
e $\begin{aligned} 17-d & =9 \\ d & =8\end{aligned}$
f $9-p=12$
$p=3$
g $3 y=9$
$y=27$
h $5 n=35$
$n=7$
i $-4 n=-20$
$n=-5$
j $\frac{x}{2}=8$
$x=4$
k $\frac{a}{4}=2$
$k=8$
( $\frac{n}{7}=7$

$$
n=1
$$

4 Solve these two-step equations. Clearly show each step in your working. (All the answers are integers.)
a $2 x+3=7$
b $3 n+5=8$
c $5 y+1=16$
d $4 n-3=9$
e $6 k-5=7$
f $9 q-4=23$
g $2-3 w=11$
h $3+5 p=3$
i $12+7 s=26$
j $12-3 x=6$
k $16-2 j=6$
l $20-9 g=2$
m $14=2 v+6$
n $30=8+2 y$

- $10=4-2 a$
p $6-x=-2$
q $-5-p=-3$
r $15=6-3 h$

5 Solve these two-step equations. (The solutions are all positive integers.)
a $\frac{5 x}{2}=5$
b $\frac{3 x}{4}=6$
c $\frac{2 t}{5}=4$
d $\frac{x+1}{2}=3$
e $\frac{a+5}{3}=3$
f $\frac{m+8}{4}=2$
g $\frac{n-1}{5}=2$
h $\frac{y-7}{2}=5$
i $\frac{x-5}{8}=3$
j $\frac{x}{2}+1=5$
k $\frac{h}{4}+3=7$
เ $\frac{a}{5}-1=5$
$m 5+\frac{x}{2}=9$
n $3+\frac{q}{4}=8$

- $9-\frac{n}{2}=5$

Myltiply each term on both sides
by the denominator.

6 Find the solutions to each of the following equations.
a $2 x+3=4$
b $3 n+6=8$
d $4 n-1=9$
e $6 k-4=7$
g $2+3 w=10$
h $3+5 p=4$
$\begin{aligned} \text { c } 5 y-1 & =3 \\ 60-4 & =2\end{aligned}$
j $12-3 x=8$
k $15-2 j=6$
l) $0-7 g=2$
m $14=2 v+3$
n $21=8+21$
o $10=5-3 y$
p $6-3 x=-2$
q $-5-2 \hat{p}=-4$
r $15=10-3 h$

$$
\begin{gathered}
3 a+2=6 \\
-2 \quad-2 \\
3 a=4 \\
\div 3 \quad \div 3 \\
a=\frac{4}{3} \text { or } 1 \frac{1}{3}
\end{gathered}
$$

7 Solve this set of equatrons, whel are either one- or two-step types.
a $2 x=7$
d $5-y=6$
e $7+3=17$
c $5 y+2=12$
f $y-5=-3$
g $y+4=1$
h $2 y-1=13$
i $-5 x=20$
j $4+5 m=8$
k $5 y=9$
( $5-2 y=9$
$m \frac{a}{2}=5$
n $\frac{3 x}{5}=9$

- $\frac{t-3}{4}=5$
p $\frac{q}{2}-1=5$
q $\frac{w}{4}+1=6$
r $\frac{3 a}{8}=2$
s $\frac{y+7}{2}=3$
t $\frac{6-a}{3}=3$
u $11-3 x=7$
v $25+2 x=11$
w $\frac{t+5}{2}=-4$
x $\frac{5-p}{4}=3$



## 都 ${ }^{\circ}$ FUN SPOT 9:02

Work out the answer to each part and write the letter for that part in the box above the correct answer.

Solve these equations.


## MATHS TERMS 9

algebraic expression

- an algebraic expression consists of one or more terms joined together by operation signs,
e.g. $a+5, x^{2}-x+4, \frac{3 m-1}{7}$
- an expression does not have an 'equals' sign like an equation equation
- a number sentence where one or more of the numbers is missing or unknown
- the unknown number is represented by a pronumeral,
e.g. $x+5=8, \frac{3 x+1}{7}=\frac{x-5}{2}$
evaluate
- to find the value of an expression, e.g. Find the value of $3 a+7$ if $a=3$
- to find the answer (usually after substitution)
grouping symbols
- the most common types are: parentheses ( ) brackets [ ] braces \{ \}
- used to 'group' a number of tenns together in an expression,
e.g. $5(x+3)$
inverse operation
- the operation that rill ryerse or 'undo'
a previous operation,
e.g. Addition is the inverse operation of subtraction; division is the inverse operation of multiplication


## pronumeral

- a symbol used to represent a number
- usually a letter such as $x$ quadratic equation
- quadratic equations have a squared pronumeral term and usually have two solutions
e.g. $x^{2}=49$ has two solutions:

$$
x=7 \text { or } x=-7
$$

solution

- the answer to a problem

OR
the method of finding the answer to a problem

- the solution to an equation is the number that makes it ane sentence
e.g. $x=3$ is the solution to $x+2=5$
solve
- find be oution or answer to a problem - or equation substicue
to roplace a pronumeral with a numeral e.g. To substitute 3 for $a$ in the expression $4 a-2$ would give:
4(3) - 2
$=12-2$
$=10$ achan


## DIAGNOSTIC TEST 9

## EQUATIONS

Each section of the test has similar items that test a certain type of question.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.
1 Solve:
9:02
a $x+11=27$
b $p-5=2$
c $5+a=3$
d $y-7=-2$

2 Solve:
c $\frac{n}{4}=8 \quad$ d $\frac{a}{3}=-2$
3 Solve these two-step equations. (Answers are integers.)
9:02
a $2 x+3=7$
b $3 m-1=8$
c $6+4 y=22$
d $4-2 a=6$

4 The answers to these equations involve fractions.
9:02
a $2 x+1=6$
b $4 y-3=4$
c $7+5 k=3$

9:02
5 Solve these two-step equations.
a $\frac{5 m}{4}=10$
b $\frac{m+2}{3}=4$
c $\frac{x}{2}+4=6 \bigcirc$ d $\frac{w-2}{5}=2$

6 Solve:


7 Solve:

$$
\begin{aligned}
& \text { a } 2(a+4)=8 \\
& \text { c } 2(2 y+1)=6
\end{aligned}
$$

8 Solve:
a $3(n+2)=2 n+7$
c $2(x+1)=5(x-5$
9 Solve:
a $x^{2}=16$
b $3 m+2=m$
a $2 a+1=a+7$
d $7 x=5 x+9$

d $3(5 m-1)=27$
b $9 p+1=7(p+3)$
d $3(2 x-1)=4(3 x-1)$
c $m^{2}=100$
d $b^{2}=3$
10 Form an equation for each number problem, letting the number be $n$, and solve it.
a The product of a number and 7 is 63 . Find the number.
b The sum of a number and 5 is 2 . What is the number?
11 Form an equation from the information with each shape, and solve it to find $x$.
a


Perimeter $=80 \mathrm{~cm}$

Area $=60 \mathrm{~cm}^{2}$
c


## ASSIGNMENT 9A Chapter review

1 Complete these arrow diagrams by filling in the boxes, showing the 'backtracking' steps.
a

$\square$
b


2 Solve these simple one-step equations
a $x+13=21$
b $m-5=3$
c $12-n=2$
d $t-9=-9$
e $6 y=-24$
f $5 x=12$
g $-8 a=-8$
h $-3 y=10$
i $\frac{x}{2}=6$
j $\frac{w}{4}=-8$
k $x-(-3)=6$
l $t-(-8)=-2$
$\mathrm{m} x^{2}=36$
n $c^{2}=81$

- $m^{2}=7$
p $b^{2}=11$

3 Solve these two-step equations.
a $2 x+3=5$
b $3 n+2=8$
c $4 n-3=17$
e $9 q-4=12$
d $2 k-5=7$
g $3+9 p=3$
i $12-3 x=7$
k $20-8 g=2$
m $27=3+2 y$
o $6-x=-6$
q $\frac{n-3}{5}=4$
$2+3 w=7$
h $12+5 s=26$


4 Solve these equations that have the pronumeral on both sides of the equation.
a $2 x+5=x+7$
b $4 y-3=2 y+5$
c $7 w+1=9 w-3$
d $2+3 p=p+8$
e $4-3 x=2 x+9$
f $7 k+5=9-3 k$

5 Solve:
a $2(x+3)=6$
b $3(n+5)=12$
c $4(n-1)=20$
d $2(3 k-4)=4$
e $5(2 q+4)=10$
f $2(5+3 w)=16$
g $6(x-2)=9$
h $3(4 x+5)=9$

6 Solve these equations.
a $2(x+3)=x+8$
b $5 a+9=6(a+$
c $4(y-3)=$
d $2(1+3 p)$
e $5(2+2=3(7-3 q)$


7
a When 5 is added to the product of a number and 7 the answer is 61 . Find the number.
b If 9 is subtracted from a number and the result is multiplied by 7 the answer is -21 . Find the number.

8 Form an equation and solve it to find $a$.


## ASSIGNMENT 9B Working mathematically

1 I purchased 15 boxes of matches. If each box contains between 47 and 54 matches, choose which of the following answers could be the total number of matches.
A 615
B 705
C 763
D 810

2 One can of food has a mass of 250 g , whereas another has a mass of 450 g . A third can has a mass of 675 g and a fourth can has the second largest mass of these four cans. The total mass of the four cans is:
A 1675 g
B 1825 g
C 1826 g
D 2125 g

3 To celebrate the birth of a new nation, 3400 people link hands to reach across the city of Sydney. If the space between bodies is 1.3 m and the width of each body is 0.5 m , how far will the people stretch?

4 The total number of votes cast in three electorates was 648 493. If in one of these electorates 249306 votes were cast and in another 198486 votes were cast, how m votes were cast in the third electorater


Some of the solids mentioned in Question 6 above can be interlocked to form this famous puzzle called the Soma Cube. Find out which ones they are.

## ASSIGNMENT 9 C Cumulative revision

1 Before they played, Sandy had 15 marbles and Luke had 10. Luke lost 5 marbles to Sandy.
a What percentage of his marbles did Luke lose?
b By what percentage did the number of Sandy's marbles increase?
2 Find the value of $x$ in each case.
3:02,


3 The probability that I will be allowed to leave before 10 am is $0 \cdot 35$. What is the probability that I won't be allowed to leave before 10 am ?

4 Calculate the area of each of these shapes.

b


5 a Expand 6(x-3).
c Expand $4 a(a+2)$.
actorise $12 a-20$.
E ctorise $6 a b+12 b$.
5:08, 5:09

6 a Convert 17000 t to kg .
b Change 7800 m to km .
c Write 800 min in hours and n intes
7 Simplify:

a $-3-4-2$
c $-3 \times 2 \times-4$
d $(-4+6) \times-2$

8 a If $40 \%$ of a fertiliser is superphosphate, how many kilograms of superphosphate are needed for 1 tonn of fertiliser?
b A real estate agent receives $4 \frac{1}{2} \%$ commission on the value of each house that she sells. How much does she receive for the sale of a house valued at $\$ 525000$ ?

9 a What is the time difference from 13:20 on Tuesday to 09:40 on Thursday?
b If I began a journey at 07:25 that took 13 hours and 40 minutes to complete, at what time did I arrive at my destination?

10 Calculate the volume of these solids, correct to two decimal places.


