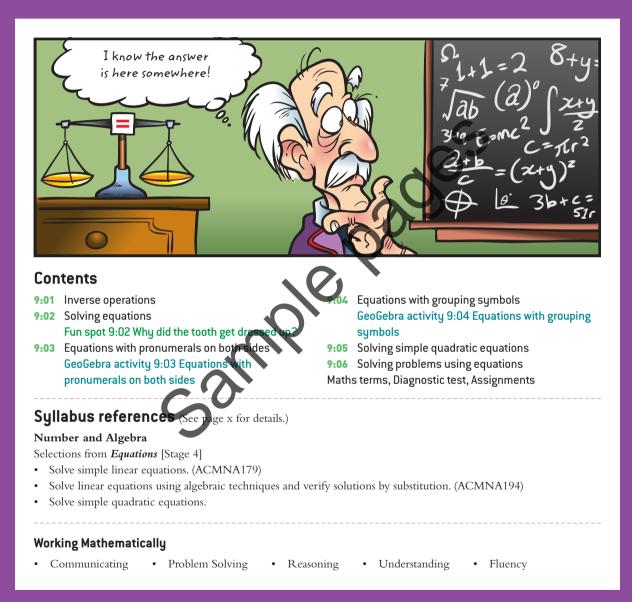
EQUATIONS





In Year 7 you were introduced to equations. An equation is a number sentence in which one (or more) of the numbers is unknown and is represented by a pronumeral, usually a letter of the alphabet.

e.g. x + 5 = 12 or 10 - 3a = 4

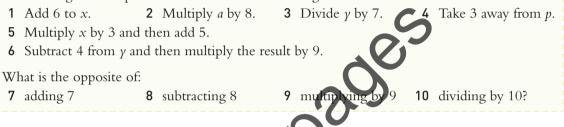
We **solve** an equation to find the value of the pronumeral.

To solve equations it is necessary to understand inverse operations.

9:01 Inverse operations

PREP QUIZ 9:01

Write an algebraic expression for each of the following sentences.



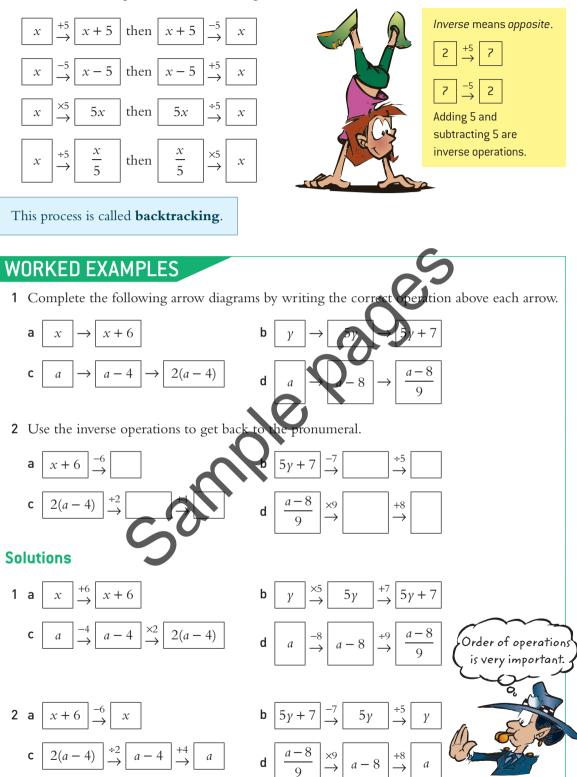
We start with a pronumeral when building algebraic expressions.

Algebraic expression	Meaning	Arrow diagram
<i>x</i> + 5	a has 5 added to it.	$x \xrightarrow{+5} x+5$
x – 5	x has 5 subtracted from it.	$x \xrightarrow{-5} x - 5$
C ³ x ² O ³	x is multiplied by 5.	$x \xrightarrow{\times 5} 5x$
$\frac{x}{5}$	x is divided by 5.	$x \xrightarrow{\div 5} \frac{x}{5}$

- Arrow diagrams can be used to show how the expression has been made.
- The operation that has caused the change is written above the arrow.

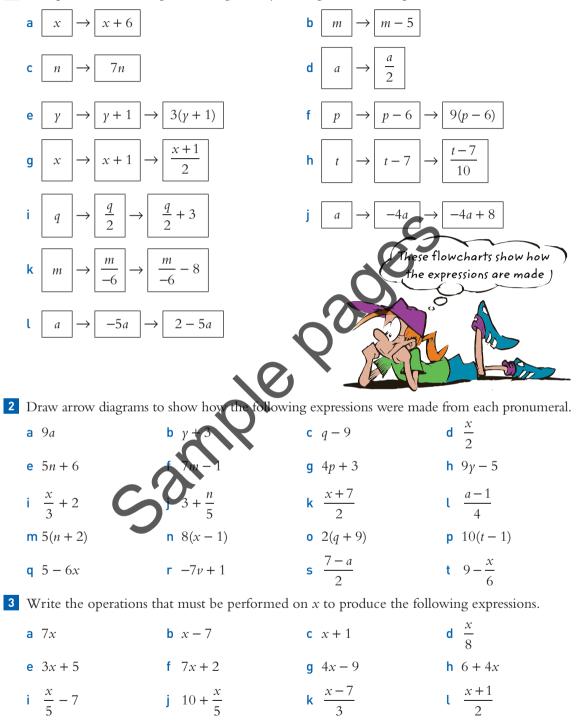
Equations are sometimes called algebraic sentences.

• When solving equations, we need to reverse the steps to get back to the pronumeral. In other words, we need to perform the **inverse** operations.



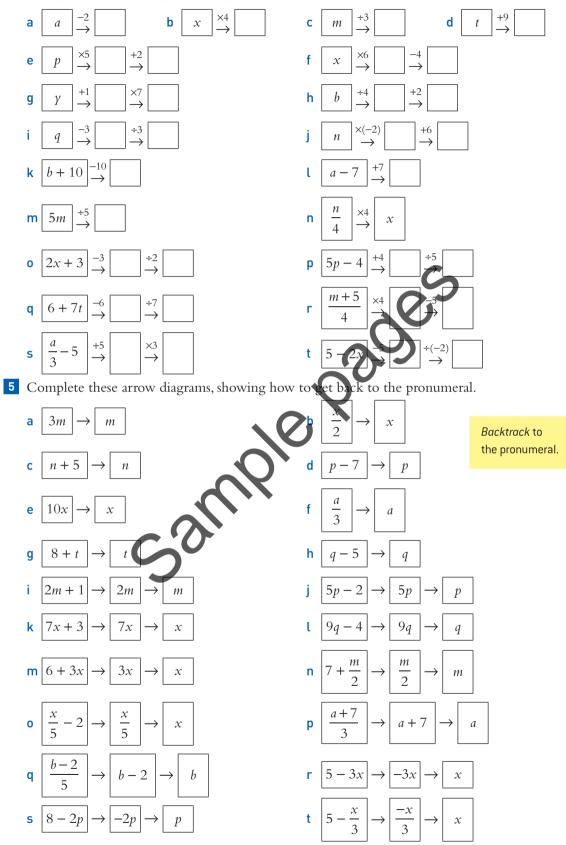
Exercise 9:01

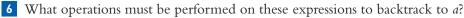
1 Complete the following arrow diagrams by writing the correct operation above each arrow.

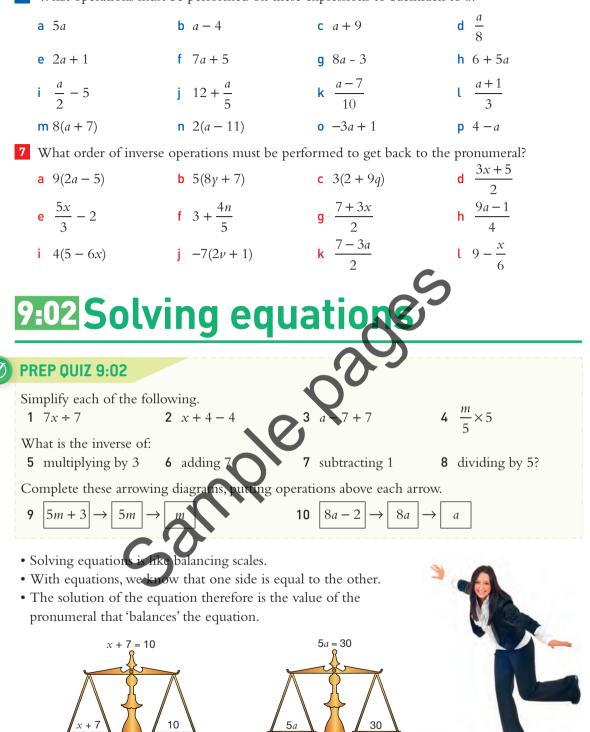


m 3(x + 7) **n** 9(x - 4) **o** -7x + 1 **p** 12 - x

4 Complete the following arrow diagrams.







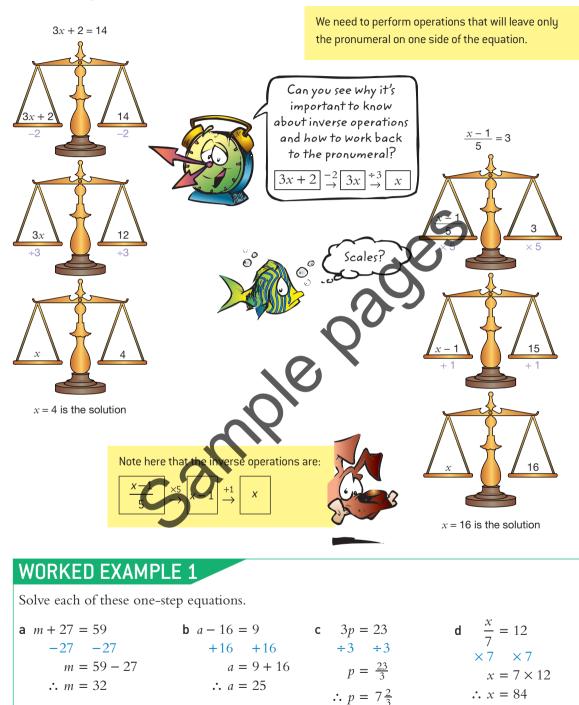
x = 3 balances the scale so x = 3 is the solution.

10

a = 6 balances the scale so a = 6 is the solution.

30

- Often, solving an equation requires us to change the equation into a simpler one. To do this we add (+), subtract (-), multiply (X) or divide (+) both sides of the equation by the same number.
- Look at the solutions of these two equations, noting that both sides remain balanced because the same operation is done to both sides.



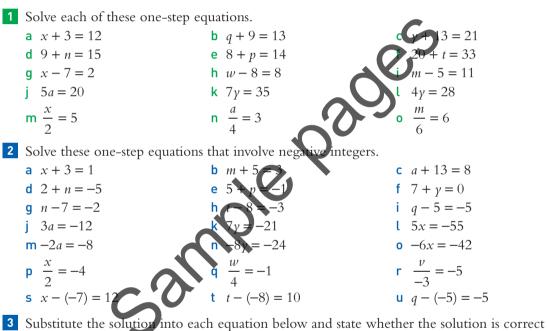
WORKED EXAMPLE 2

Solve each of these two-step equations.

a $2a + 5 = 7$	b $\frac{x+3}{5} = 2$	c $\frac{p}{2} - 6 = 15$	d $10 - 3m = 25$
-5 -5	> ×5 ×5	3 + 6 + 6	-10 -10
2a = 2	x + 3 = 10	$\frac{p}{2} = 21$	-3m = 15
÷2 ÷2	-3 -3	3	$\div -3 \div -3$
$\therefore a = 1$	$\therefore x = 7$	×3 ×3	$\therefore m = -5$
		$\therefore p = 63$	

Exercise 9:02





or not.

01 1100.			
a <i>x</i> + 17 = 39	b 9 + m = 15	c $q - 13 = 16$	d $n - 28 = 17$
x = 56	m = 7	q = 29	n = 11
e $17 - d = 9$	f $9 - p = 12$	g $3\gamma = 9$	h $5n = 35$
d = 8	p = 3	$\gamma = 27$	n = 7
i $-4n = -20$	j $\frac{x}{2} = 8$	k $\frac{a}{4} = 2$	$l \frac{n}{7} = 7$
n = -5	x = 4	k = 8	n = 1

4 Solve these two-step equations. Clearly show each step in your working. (All the answers are integers.)

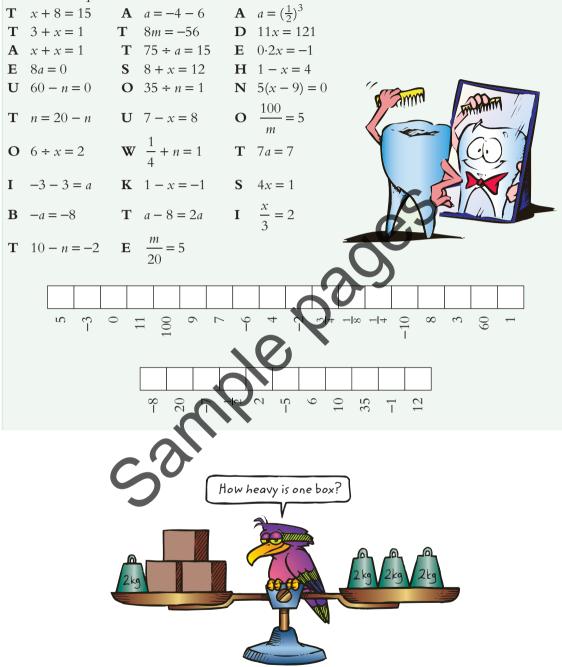
	a $2x + 3 = 7$		+5 = 8	c $5\gamma + 1 = 16$
	d $4n - 3 = 9$		-5 = 7	f $9q - 4 = 23$
	g $2 - 3w = 11$		+5p=3	i $12 + 7s = 26$
	j $12 - 3x = 6$		-2j = 6	$1 \ 20 - 9g = 2$
	m $14 = 2\nu + 6$		$= 8 + 2\gamma$	o $10 = 4 - 2a$
	p $6 - x = -2$	q -5	-p = -3	r $15 = 6 - 3h$
5	Solve these two-st positive integers.)	ep equations. (The	solutions are all	3x
		3x	2t	$\frac{3x}{5} = 6$
	2	b $\frac{3x}{4} = 6$	5	$\times 5 \times 5$ 3x = 30
	d $\frac{x+1}{2} = 3$	e $\frac{a+5}{3} = 3$	$f \frac{m+8}{4} = 2$	\div 3 \div 3 x = 10
	g $\frac{n-1}{5} = 2$	h $\frac{\gamma-7}{2} = 5$	i $\frac{x-5}{8} = 3$	S
	j $\frac{x}{2} + 1 = 5$	k $\frac{h}{4} + 3 = 7$	$l \frac{a}{5} - 1 = 5$	
	$m 5 + \frac{x}{2} = 9$	n $3 + \frac{q}{4} = 8$	• $9 - \frac{n}{2} = 5$	Multiply each term on <i>both sides</i> by the denominator.
6	Find the solutions	to each of the follo	wing equations	
	a $2x + 3 = 4$	b $3n + 6 = 8$	c $5\gamma + 1 = 3$	3a + 2 = 6
	d $4n - 1 = 9$		6a - 4 = 2	-2 -2 -2
	g $2 + 3w = 10$		12 + 7s = 20	3a = 4
	j 12 - 3x = 8	÷	1 20 - 7g = 2	÷3 ÷3
	m $14 = 2\nu + 3$	· · ·	$10 = 5 - 3\gamma$	$a = \frac{4}{3}$ or $1\frac{1}{3}$
	p $6 - 3x = -2$	q $-5 - 2p = -4$	r $15 = 10 - 3h$	
7	Solve this set of ea	puations, which are	either one- or two-step	types.
			c $5\gamma + 2 = 12$	
	d $5 - \gamma = 6$	e 7x + 3 = 17	f $y - 5 = -3$	Opposite operations
	g $\gamma + 4 = 1$	h $2\gamma - 1 = 13$	-5x = 20	are the key.
	j $4 + 5m = 8$	k $5\gamma = 9$	$5 - 2\gamma = 9$	
	m $\frac{a}{2} = 5$	n $\frac{3x}{5} = 9$	• $\frac{t-3}{4} = 5$	
	p $\frac{q}{2} - 1 = 5$	q $\frac{w}{4} + 1 = 6$	r $\frac{3a}{8} = 2$	Car ter
		$t \frac{6-a}{3} = 3$		
	v $25 + 2x = 11$	$w \frac{t+5}{2} = -4$	$\frac{5-p}{4} = 3$	

畿 FUN SPOT 9:02

WHY DID THE TOOTH GET DRESSED UP?

Work out the answer to each part and write the letter for that part in the box above the correct answer.

Solve these equations.



MATHS TERMS 9

algebraic expression

 an algebraic expression consists of one or more terms joined together by operation signs,

e.g. a + 5, $x^2 - x + 4$, $\frac{3m - 1}{7}$

• an expression does *not* have an 'equals' sign like an equation

equation

- a number sentence where one or more of the numbers is missing or unknown
- the unknown number is represented by a pronumeral, 3x+1 x-5

e.g.
$$x + 5 = 8$$
, $\frac{5x + 1}{7} = \frac{x - 5}{2}$

evaluate

- to find the value of an expression,
 e.g. Find the value of 3a + 7 if a = 3
- to find the answer (usually after substitution)

grouping symbols

- the most common types are: parentheses () brackets [] braces { }
- used to 'group' a number of terr together in an expression,
 e.g. 5(x + 3)

inverse operation

- the operation that will reverse or 'undo' a previous operation,
 - e.g. Addition is the inverse operation of subtraction; division is the inverse operation of multiplication

pronumeral

- a symbol used to represent a number
- usually a letter such as *x*

quadratic equation

• quadratic equations have a squared pronumeral term and usually have two solutions

e.g. $x^2 = 49$ has two solutions:

x = 7 or x = -7

solution

 the answer to a problem OR the method of finding the answer to

a problem

- the solution to an equation is the number that makes it a true sentence
- e.g. x = 3 is the solution to x + 2 = 5solve
- find the solution or answer to a problem or equation

substitute

- to replace a pronumeral with a numeral e.g. To substitute 3 for *a* in the expression 4a - 2 would give:
 - 4(3) 2= 12 - 2 = 10

DIAGNOSTIC TEST 9

EQUATIONS

Each section of the test has similar items that test a certain type of question.

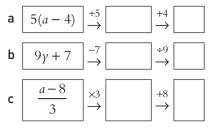
Errors made will indicate areas of weakness.

Each weakness should be treated by going back to the section listed.

1 Solve: a $x + 11 = 27$ b $p - 5 = 2$ c $5 + a = 3$ d $y - 7 = -2$	9:02
2 Solve: a $6m = 54$ b $3x = 5$ c $\frac{n}{4} = 8$ d $\frac{a}{3} = -2$	9:02
3 Solve these two-step equations. (Answers are integers.) a $2x + 3 = 7$ b $3m - 1 = 8$ c $6 + 4y = 22$ d $4 - 2a = 6$	9:02
4 The answers to these equations involve fractions. a $2x + 1 = 6$ b $4y - 3 = 4$ c $7 + 5k = 3$ d $9 - 2t = 6$	9:02
5 Solve these two-step equations. a $\frac{5m}{4} = 10$ b $\frac{m+2}{3} = 4$ c $\frac{x}{2} + 4 = 6$ d $\frac{w-2}{5} = 2$	9:02
6 Solve: a $2a + 1 = a + 7$ b $3m + 2 = m - 6$ d $7x = 5x + 9$	9:03
7 Solve: a $2(a + 4) = 8$ b $6(m - 3) = 12$ c $2(2y + 1) = 6$ d $3(5m - 1) = 27$	9:04
8 Solve: a $3(n+2) = 2n+7$ c $2(x+1) = 5(x-5)$ b $9p+1 = 7(p+3)$ d $3(2x-1) = 4(3x-1)$	9:04
9 Solve: a $x^2 = 16$ b $c^2 = 9$ c $m^2 = 100$ d $b^2 = 3$	9:05
10 Form an equation for each number problem, letting the number be n, and solve it.a The product of a number and 7 is 63. Find the number.b The sum of a number and 5 is 2. What is the number?	9:06
11 Form an equation from the information with each shape, and solve it to find x. a 6x cm = 4x cm 4x cm Perimeter = 80 cm b (3x+5) cm $Area = 60 \text{ cm}^2$ C $(x+30)^\circ$ $(x+30)^\circ$ $(x+30)^\circ$	9:06

ASSIGNMENT 9A Chapter review

1 Complete these arrow diagrams by filling in the boxes, showing the 'backtracking' steps.



- **2** Solve these simple one-step equations
 - **a** x + 13 = 21**b** m - 5 = 3**c** 12 - n = 2**d** t - 9 = -9e 6v = -24f 5x = 12**h** $-3\gamma = 10$ **q** -8a = -8i $\frac{x}{2} = 6$ j $\frac{w}{4} = -8$ t - (-8) = -2k x - (-3) = 6**n** $c^2 = 81$ $m x^2 = 36$ **o** $m^2 = 7$ **p** $b^2 = 11$

3 Solve these two-step equations.

a 2x + 3 = 5**b** 3n + 2 = 8**c** 4n - 3 = 17**d** 2k - 5 = 7e 9q - 4 = 12f 2 + 3w = 7**g** 3 + 9p = 3h 12 + 5s =i 12 - 3x = 716 k 20 - 8g = 214 $m 27 = 3 + 2\gamma$ **o** 6 - x = -6 $q \frac{n-3}{5} = 4$ s $\frac{x}{2} - 7 = 5$ +7 = 5

- 4 Solve these equations that have the pronumeral on both sides of the equation.
 - **a** 2x + 5 = x + 7 **b** 4y - 3 = 2y + 5 **c** 7w + 1 = 9w - 3 **d** 2 + 3p = p + 8 **e** 4 - 3x = 2x + 9**f** 7k + 5 = 9 - 3k
- **5** Solve:
 - **a** 2(x + 3) = 6**b** 3(n + 5) = 12**c** 4(n 1) = 20**d** 2(3k 4) = 4**e** 5(2q + 4) = 10**f** 2(5 + 3w) = 16**q** 6(x 2) = 9**h** 3(4x + 5) = 9
- **6** Solve these equations.
 - **a** 2(x+3) = x+8
 - **b** 5a + 9 = 6(a + 1)
 - **c** 4(y-3) = 2(y+5)
 - **d** 2(1+3p) = 5(p+8)
 - e 5(2+q) = 3(7-3q)f 2(4-3x) = 3(2x+9)
- 7 White an equation for each problem and solve it. Let the number be *x*.
 - **a** When 5 is added to the product of a number and 7 the answer is 61. Find the number.
 - b If 9 is subtracted from a number and the result is multiplied by 7 the answer is -21. Find the number.
- 8 Form an equation and solve it to find *a*.

$$6 \text{ cm}$$
 Area = 270 cm²
(8*a* + 5) cm

ASSIGNMENT 9B Working mathematically

- 1 I purchased 15 boxes of matches. If each box contains between 47 and 54 matches, choose which of the following answers could be the total number of matches.
 A 615 B 705 C 763 D 810
- 2 One can of food has a mass of 250 g, whereas another has a mass of 450 g. A third can has a mass of 675 g and a fourth can has the second largest mass of these four cans. The total mass of the four cans is:
 A 1675 g B 1825 g C 1826 g D 2125 g
- 3 To celebrate the birth of a new nation, 3400 people link hands to reach across the city of Sydney. If the space between bodies is 1.3 m and the width of each body is 0.5 m, how far will the people stretch?
- 4 The total number of votes cast in three electorates was 648 493. If in one of these electorates 249 306 votes were cast and in another 198 486 votes were cast, how many votes were cast in the third electorate?

5 Sophia is 28 years younger than Alec.Roman is one-quarter the age of Sophia.Maria is 18 years older than Roman.If Maria is 28, how old is Alec?



6 When three cubes are joined at their faces they can only form two different solids. These are shown here. How many different solids can be formed by joining



Some of the solids mentioned in Question 6 above can be interlocked to form this famous puzzle called the Soma Cube. Find out which ones they are.

four cubes

ASSIGNMENT 9C Cumulative revision

	Before they played, Sandy had 15 marbles and Luke had 10. Luke lost 5 marbles to Sandy.a What percentage of his marbles did Luke lose?b By what percentage did the number of Sandy's marbles increase?	4:06
2	Find the value of x in each case. a 2 cm 1.5 cm b 132° x°	3:02, 1:08
	The probability that I will be allowed to leave before 10 am is 0.35. What is the probability that I won't be allowed to leave before 10 am?	6:03
4	Calculate the area of each of these shapes. $a \rightarrow 6 \cdot 8 \text{ cm}$ $b \rightarrow 6 \cdot 6 \text{ cm}$ $g \rightarrow 8 \text{ cm}$ $g \rightarrow 15 \text{ cm}$ $g \rightarrow 15 \text{ cm}$	8:04
	a Expand $6(x-3)$.b Pactorise $12a - 20$.c Expand $4a(a+2)$.c Eactorise $6ab + 12b$.	5:08, 5:09
	 a Convert 17000t to kg. b Change 7800m to km. c Write 800min in hours and numetes 	1:11
	Simplify: a $-3 - 4 - 2$ b $2 - 4 - 8$ c $-3 \times 2 \times -4$ d $(-4 + 6) \times -2$	1:12
	 a If 40% of a fertiliser is superphosphate, how many kilograms of superphosphate are needed for 1 tonne of fertiliser? b A real estate agent receives 4¹/₂% commission on the value of each house that she sells. How much does she receive for the sale of a house valued at \$525 000? 	4:06
	a What is the time difference from 13:20 on Tuesday to 09:40 on Thursday?b If I began a journey at 07:25 that took 13 hours and 40 minutes to complete, at what time did I arrive at my destination?	8:08
10	Calculate the volume of these solids, correct to two decimal places. a 6 cm b 8.6 cm 4.6 cm 10.4 cm	8:06