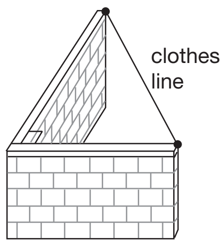
Student name: ....

Class: .... Date: ....

3:01 Investigating right-angled triangles

One part of a garden is fenced by two walls at right angles. One wall is 8 m long, and the other is 15 m.



The owner wants to run a clothesline from the end of one wall to the end of the other.

1 Will the clothesline be longer than 15 m? Why?

____

____

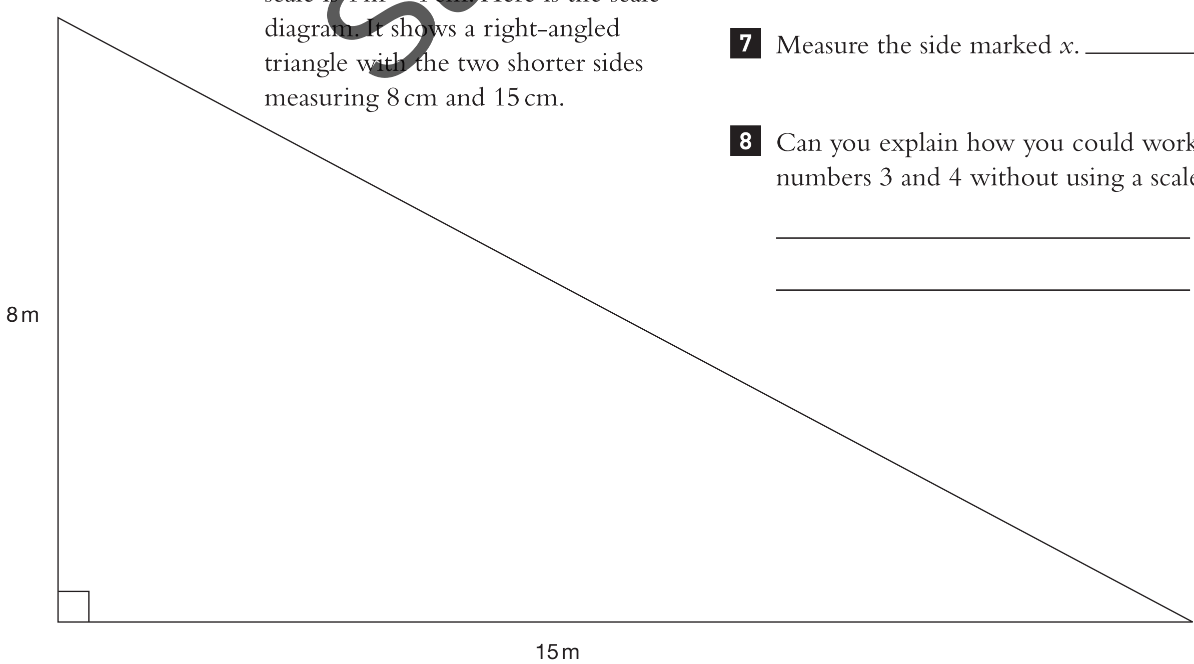
2

2 Take a guess at how long it might be.

____

1

To estimate how long the line will be, the owner draws a scale diagram. The scale is 1 m = 1 cm. Here is the scale diagram. It shows a right-angled triangle with the two shorter sides measuring 8 cm and 15 cm.



3 Measure the third side of the triangle. What is its length, in centimetres? What length does this represent, in metres?

____

____

2

4 Do you think the line will need to be longer than the estimate that the owner gets from the scale diagram? Explain your answer.

____

____

2

5 Mathematicians can calculate this length exactly without drawing a scale diagram. Investigate.
Hint: How do you get the number 289 from working out 8^2 and 15^2? Calculate $\sqrt{289}$.

____

1

6 Make an exact scale drawing of a right-angled triangle with perpendicular sides 4 cm and 3 cm. Mark the unknown side length *x*.

1

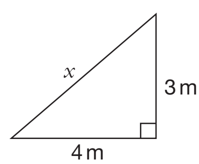
7 Measure the side marked *x*. ______________________

1

8 Can you explain how you could work out *x* from the numbers 3 and 4 without using a scale drawing?

____

____



1

Result /11

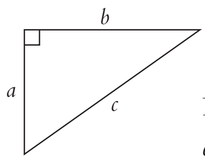
3 PYTHAGORAS' THEOREM 1

Student name: ……………………………………………………………

Class: ……………………… Date: ……………………………

3:02 Pythagoras' theorem: Calculating the hypotenuse

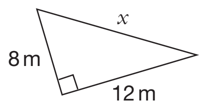
In this diagram of a right-angled triangle, the two sides at right-angles are marked a and b. The **hypotenuse** (side opposite the right-angle) is labelled c.



Pythagoras' theorem
$c^2 = a^2 + b^2$

Example:

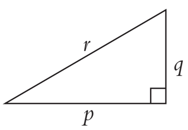
Calculate the length of the side marked x.



$x^2 = 8^2 + 12^2$
$= 64 + 144$
$= 208$
$x = \sqrt{208} = 14{\cdot}42\,\text{m}$ (4 sig. fig.)

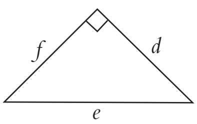
1 Write the relationship for each of these triangles using Pythagoras' theorem.

a



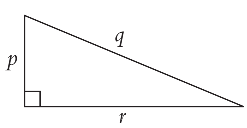
____

b



____

c



____

3

2 Complete the working using Pythagoras' theorem.

a $c^2 = 7^2 + 24^2$

$c^2 = 49 + 576$

$c^2 = 625$

$c =$ ______

b $c^2 = 5^2 + 9^2$

____

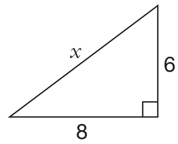
____

____

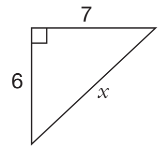
2

3 Use Pythagoras' theorem to calculate the length of the unknown marked side in each right-angled triangle. All measurements are in centimetres. Round answers to one decimal place, where necessary.

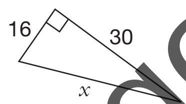
a



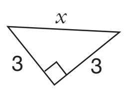
b



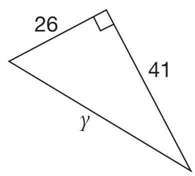
c



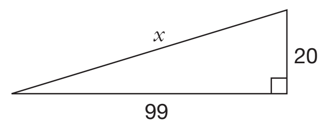
d



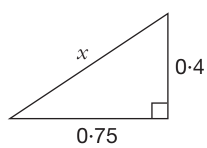
e



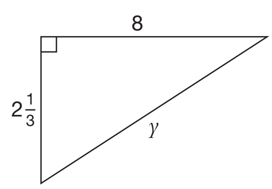
f



g



h



8

Result /13

Student name: ....

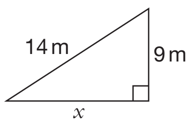
Class: .... Date: ....

3:03 Pythagoras' theorem: Calculating one of the shorter sides

Pythagoras' theorem can also be used to calculate one of the two shorter (or perpendicular) sides (a side that is not the hypotenuse). This involves subtraction.

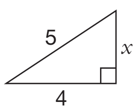
Example:

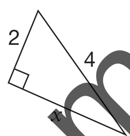
Calculate the length of the side marked x.

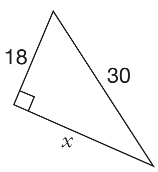


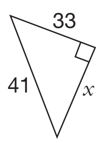
$$14^2 = x^2 + 9^2$$
$$196 = x^2 + 81$$
$$x^2 = 196 - 81$$
$$x^2 = 115$$
$$x = \sqrt{115} = 10{\cdot}72\,\text{m (4 sig. fig.)}$$

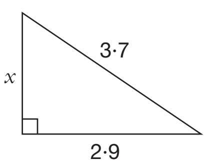
1 Use Pythagoras' theorem to calculate the length of the unknown marked side in each right-angled triangle. All measurements are in centimetres. Round answers to one decimal place, where necessary.

a


b


c


d


e


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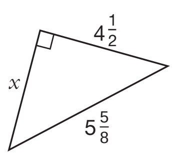
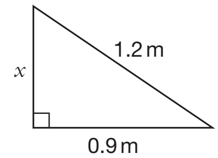


6

For Questions **2–4**, answer correct to two significant figures.

2 A student wrote down this working when calculating x.



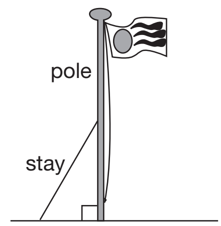
$$(1{\cdot}2)^2 = x + (0{\cdot}9)^2$$
$$1{\cdot}44 = x + 0{\cdot}81$$
$$x = 1{\cdot}44 - 0{\cdot}81$$
$$x = 0{\cdot}63$$

a Explain what the mistake is in the working.

b Calculate the correct answer.

2

3 A flagpole is supported by a stay that is 3·7 m long. The stay is attached to the ground at a point that is 1·4 m from the base of the pole.

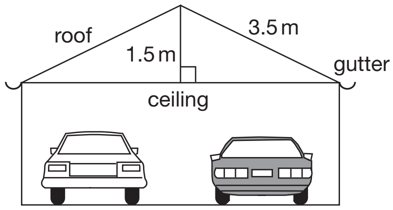


a Draw a right-angled triangle, with measurements on two sides and x on the third side, to represent this information.

b Calculate how far up the pole the stay is fastened.

2

4 The top of a garage roof is 1·5 m above the ceiling. The distance from the top of the roof to each gutter is 3·5 m. Calculate the width of the garage.



2

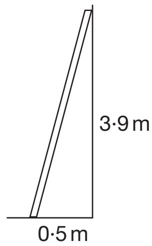
Result /12

Student name: ....

Class: .... Date: ....

3:04 Applications of Pythagoras' theorem

1 A length of board has been placed against a wall on an angle to prevent it from falling over. It reaches 3·9 m up the wall, and the bottom of the board rests on the floor 0·5 m from the wall. Use Pythagoras' theorem to calculate the length of the board, correct to two decimal places.



1

2 A pole-vaulter has left a pole standing against the wall of the changing rooms in a stadium. The pole reaches 3·6 m up the wall and rests at a point that is 0·4 m from the base of the wall.

a Draw a diagram to represent this information.

b Calculate the length, in metres, of the pole, correct to three decimal places.

2

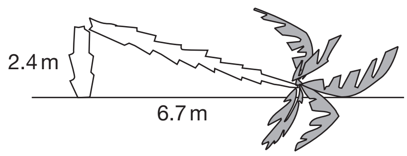
3 A ramp runs in a straight line from a point that is 6·51 m from a building to a point on the building that is 1·28 m above the ground.

a Draw a diagram to represent this information.

b Use Pythagoras' theorem to calculate the length of the ramp, correct to two decimal places.

2

4 A coconut palm has blown over after a tropical cyclone. The top part snapped off at a point 2·4 m off the ground and is resting on the ground at a point that is 6·7 m from the base of the palm. Use Pythagoras' theorem to calculate the height of the palm before it was blown over.



2

5 To protect against damage from an oncoming storm, Lucy runs some masking tape diagonally across a window from the bottom left corner to the top right corner. She uses 2·8 m of tape. The window is 1·6 m long. Explain, using a diagram and some calculations, how she could calculate the height of the window.

2

6 Here is some information about a kiwi-fruit orchard.

- It is rectangular.
- One side measures 28 m.
- The distance between opposite corners is 35 m.

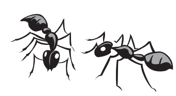
a Draw a diagram to represent this information.

b Calculate the length of a fence needed to completely enclose the orchard.

2

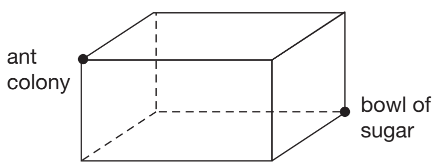
INVESTIGATION — THE ANTS AND THE SUGAR BOWL

A room in a house measures 6 m by 3 m. The height of the room is 2 m.



Some ants have discovered a bowl of sugar on the floor in one corner of the room. The entrance to their colony is in the opposite corner of the the roof.

What is the least distance they will have to travel along the roof, floor and/or walls from the colony to the bowl of sugar?



2

Result /13

Student name:

....

Class: .... Date: ....

3:05 Pythagorean triads and right-angled triangles

The converse of Pythagoras' theorem is that if three sides of a triangle (*d*, *e* and *f*) fit the relationship $d^2 = e^2 + f^2$ then the triangle must be right-angled.

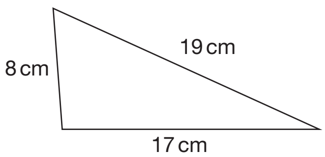
Example:

Is a triangle with sides, 8 cm, 17 cm and 19 cm right-angled?

Is the square of the longest side equal to the sum of the squares of the other two sides?

$19^2 = 361$
$8^2 + 17^2 = 64 + 289 = 353$

The triangle is not-right-angled.



Usually the sides in a right-angled triangle do not work out exactly to whole numbers. There are some exceptions, and these are called **Pythagorean triads**.

Some examples are {3, 4, 5}, {5, 12, 13}, {7, 24, 25} and multiples of these.

1 Here are the side lengths of some triangles. Use a calculator and Pythagoras' theorem to decide which groups could be measurements from a right-angled triangle. For each set answer *yes* or *no*.

a 5, 6, 7 Does $7^2 = 5^2 + 6^2$? ______
b 6, 8, 10 Does $10^2 = 6^2 + 8^2$? ______
c 2, 2, 4 Does $4^2 = 2^2 + 2^2$? ______
d 3, 3, 3 Does $3^2 = 3^2 + 3^2$? ______
e 8, 15, 17 Does $17^2 = 8^2 + 15^2$? ______
f 33, 56, 65 Does $65^2 = 33^2 + 56^2$? ______
g 26, 24, 10 Does $26^2 = 10^2 + 24^2$? ______
h 4, 5, 3 Does $5^2 = 3^2 + 4^2$? ______
i 99, 20, 101 Does $101^2 = 20^2 + 99^2$? ______

9

2 Some Pythagorean triads begin with an odd number:

{7 24 25} {9 40 41} {11 60 61}

Try squaring the first number to see if there is a pattern linking it to the other two numbers of the triad. Describe the pattern in your own words.

____

____

1

3 Write as many Pythagorean triads as you can using any *three* of the numbers {29, 99, 21, 20, 101}

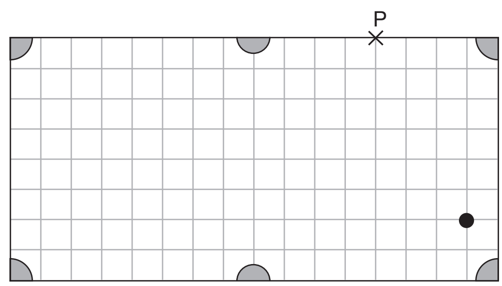
____

____

2

INVESTIGATION THE SNOOKER TABLE

The diagram shows a snooker table. It measures 4 m by 2 m, so each unit on the square grid represents 0·25 m.



A person hits the ball (marked by the dot) firmly towards point *P*, where it bounces symmetrically off the cushion. It continues in this way until it reaches one of the six holes.

1 Add the path of the ball to the diagram.
2 Use Pythagoras' theorem to calculate the *total* distance travelled by the ball, correct to two decimal places.

3 How far does the ball travel in metres?

2

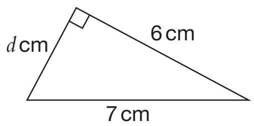
Result /14

Student name: ....

Class: .... Date: ....

3:06 Irrational numbers (Surds)

Some numbers that are the results of Pythagorean calculations cannot be expressed as exact decimals. Such numbers are called **irrational**. Examples include $\sqrt{2}, \sqrt{7}, \sqrt{13}$. In contrast, **rational numbers** include all fractions, and both terminating decimals, like 0·314 and recurring decimals, like $0.\dot{2}\dot{7} = 0.272727\ldots$

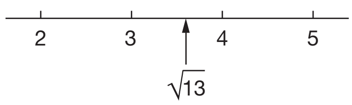


In this right-angled triangle:

$$\begin{aligned} d^2 &= 7^2 - 6^2 \\ &= 49 - 36 \\ &= 13 \end{aligned}$$

The *exact* answer for d is $\sqrt{13}$.

On a number line $\sqrt{13}$ lies in between 3 and 4.



A calculator can give an approximate value for $\sqrt{13}$ (it is about 3·606), as shown on the number line.

1 For each number, state whether is it *rational* or *irrational*.

a $\sqrt{11}$ ______________

b 0·81 ______________

c $\frac{3}{16}$ ______________

d 8 ______________

e $\sqrt{196}$ ______________

f $\frac{6}{11}$ ______________

g $\sqrt{26}$ ______________

h $0.\dot{7}$ ______________

8

2 Which parts in Question **1** show a surd?

____

2

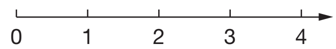
3 Between which two consecutive whole numbers does each of these surds lie?

a $\sqrt{2}$ ______________

b $\sqrt{17}$ ______________

2

4 Plot $\sqrt{5}$ on the number line below.



1

5 Arrange the set $\left\{\sqrt{13}, 5, \sqrt{11}, 4, \sqrt{17}, 3\right\}$ from smallest to largest.

____

1

6 Evaluate these surd expressions.

a $\sqrt{8} \times \sqrt{2}$ ______________

b $\sqrt{8} \div \sqrt{2}$ ______________

2

7 Evaluate:

a $(\sqrt{5} + \sqrt{3}) \times (\sqrt{5} - \sqrt{3})$

b $(\sqrt{8} - \sqrt{2}) \times (\sqrt{8} + \sqrt{2})$

c $(\sqrt{13} + 3)(\sqrt{13} - 3)$

d $(7 - \sqrt{5})(7 + \sqrt{5})$

4

8 Use your answers to Question **7** to predict the answer to $(\sqrt{11} + \sqrt{4}) \times (\sqrt{11} - \sqrt{4})$.

____

1

Result /21